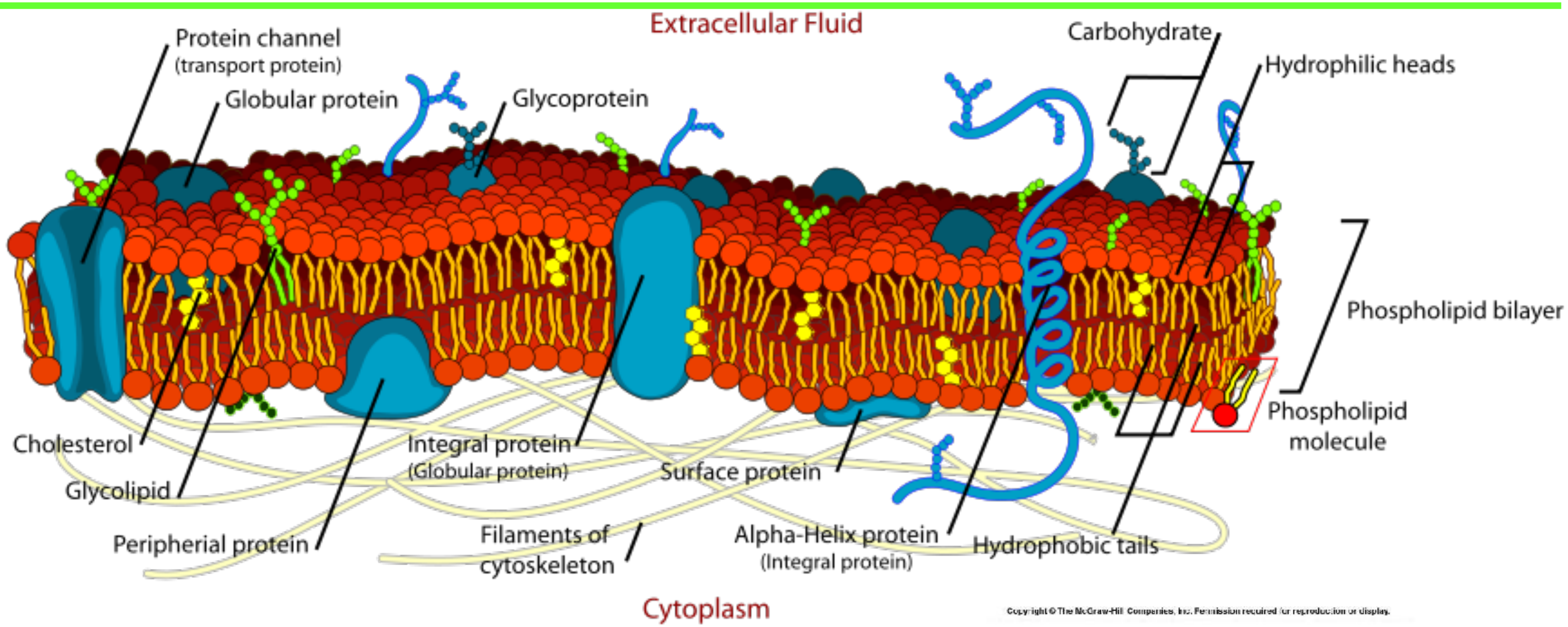
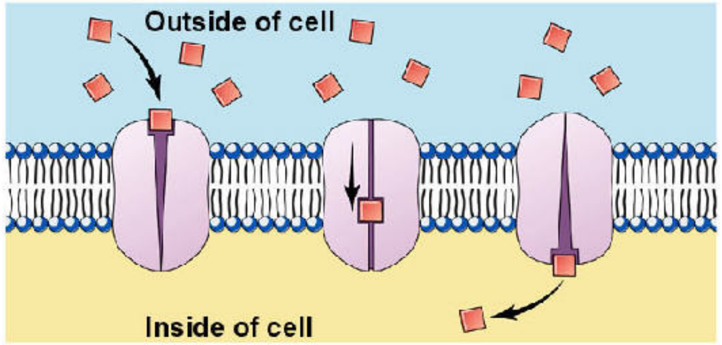
Membrane



Facilitated Diffusion



HH Membrane model

- Nernst equation at 37C

$$E_{ion} = -\frac{RT}{z_k F}\ln\frac{c_i}{c_o}$$

$$= -61\times\log_{10}\frac{c_i}{c_o}[mv]$$

$$E_{Na} = -\frac{RT}{z_k F}\ln\frac{c_i}{c_o} = -61\times\log_{10}\frac{15}{150}[mv] = +61mv$$

$$E_{K} = -\frac{RT}{z_k F}\ln\frac{c_i}{c_o} = -61\times\log_{10}\frac{150}{5.5}[mv] = -88mv$$

$$E_{Cl} = -\frac{RT}{z_k F}\ln\frac{c_i}{c_o} = -61\times\log_{10}\frac{9}{125}[mv] = -70mv$$

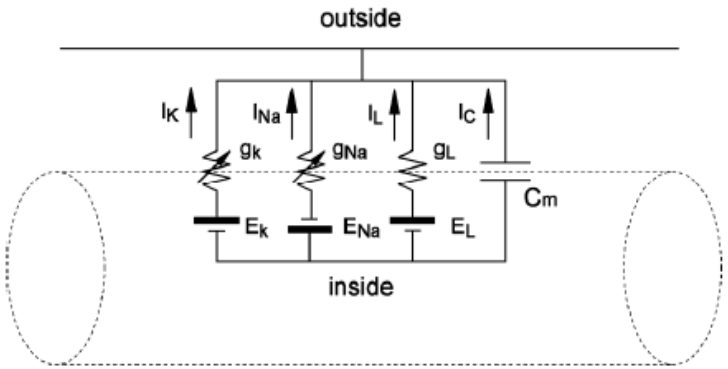
- Donna equilibrium

$$\frac{c_{o,K}}{c_{i,K}} = \frac{c_{o,Na}}{c_{i,Na}} = \frac{c_{i,Cl}}{c_{o,Cl}}$$

- Goldman-Hodgkin-Katz equation

$$V_m = -61\times\log_{10}\frac{P_K c_{i,K} + P_{Na}c_{i,Na} + P_{Cl}c_{o,Cl}}{P_K c_{o,K} + P_{Na}c_{o,Na} + P_{Cl}c_{i,Cl}}[mv]$$

$$= -70mv$$

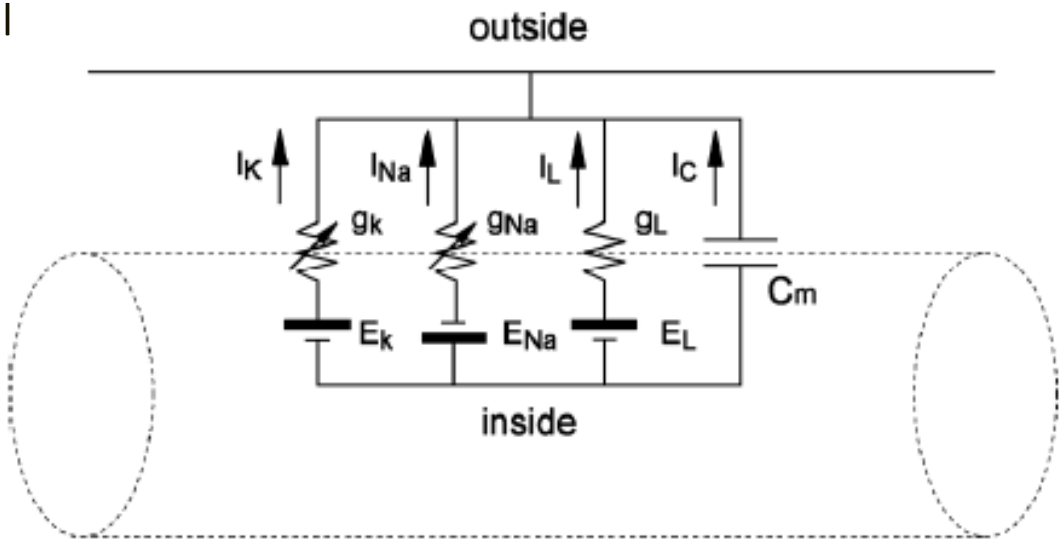


HH Membrane model

Passive model

$$i_m = \frac{V_m}{r_m} + c_m\left(\frac{\partial V_m}{\partial t}\right)$$

HH model



$$I_m = C_m \frac{dV}{dt} + I_K + I_{Na} + I_L$$

$$I_m = C_m \frac{dV}{dt} + g_K(V,t)(V - E_K) + g_{Na}(V,t)(V - E_{Na}) + g_L(V - E_L)$$

$$g_{\mathrm{K}} = 0.415\,\mathrm{mS/cm^2}, \quad g_{\mathrm{Cl}} = 0.582\,\mathrm{mS/cm^2}, \quad g_{\mathrm{Na}} = 0.010\,\mathrm{mS/cm^2}$$

HH Membrane model

$$I_m = C_m \frac{dV}{dt} + I_K + I_{Na} + I_L$$

$$I_m = C_m \frac{dV}{dt} + g_K(V,t)(V - E_K) + g_{Na}(V,t)(V - E_{Na}) + g_L(V - E_L)$$

Relationship to passive model

$$\frac{V_m}{r_m} = g_{Na} m^3(v,t) h(v,t)(V_m - E_{Na}) + g_k n^4(v,t)(V_m - E_K) + g_L(V - E_L)$$

$$\frac{dm}{dt} = [-(\alpha_m + \beta_m)m + \alpha_m]k$$

$$\frac{dh}{dt} = [-(\alpha_h + \beta_h)h + \alpha_h]k$$

$$\frac{dn}{dt} = [-(\alpha_n + \beta_n)n + \alpha_n]k$$

$$k = 3^{0.1T - 0.63}$$

Ionic current – V_m/r_m

Coefficients g's are the maximum conductance for sodium, potassium, and leakage per cm^2 respectively

n, h, m are probabilities that reduces the maximum conductance

Hodgkin-Huxley Equation

- Used to model changes in membrane conductance due to changes in membrane voltage
- Sodium current = C'*m^3h
- Potassium current =C''*n^4
  - m – sodium activation
  - h – sodium channel inactivation
  - n – potassium channel activation
  - C – constants for specific cell
- m with shortest time constant, depolarizes the cell
- h, n, with longer time constants, repolarize the cell

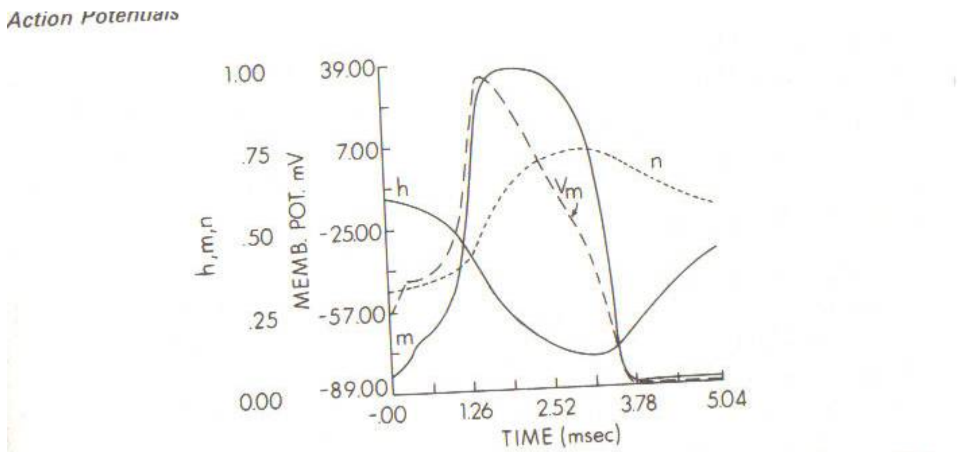


Figure 4.17. Computed temporal variation in $m(t)$, $n(t)$, $h(t)$, and $V_m(t)$ for a membrane action potential, corresponding to a depolarizing current pulse of 75 μA starting at $t = 0$ and terminating at $t = 0.25$ msec. Resting $V_m = -60$ mV. (From Y. Palti, Analysis and reconstruction of axon membrane action potential, in *Biophysics and Physiology of Excitable Membranes*, W. J. Adelman, Jr., ed., Van Nostrand Reinhold Co., New York, 1971.)

HH Membrane model

$$I_m = C_m \frac{dV}{dt} + g_K(V,t)(V - E_K) + g_{Na}(V,t)(V - E_{Na}) + g_L(V - E_L)$$

$$g_K(V,t) = n^4 \bar{g}_K$$

$$g_{Na}(V,t) = m^3 h \bar{g}_{Na}$$

$$\frac{dn}{dt} = \alpha_n(1-n) - \beta_n n, \quad n_\infty = \frac{\alpha_n}{\alpha_n + \beta_n}, \quad \tau_n = \frac{1}{\alpha_n + \beta_n}$$

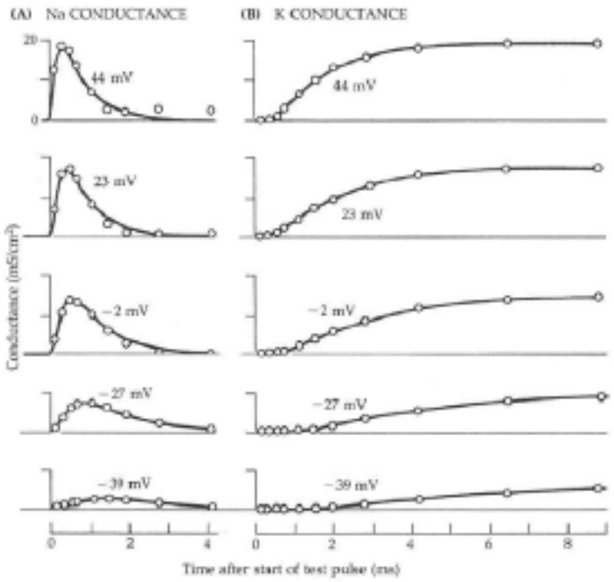
$$\frac{dm}{dt} = \alpha_m(1-m) - \beta_m m, \quad m_\infty = \frac{\alpha_m}{\alpha_m + \beta_m}, \quad \tau_m = \frac{1}{\alpha_m + \beta_m}$$

$$\frac{dh}{dt} = \alpha_h(1-h) - \beta_h h, \quad h_\infty = \frac{\alpha_h}{\alpha_h + \beta_h}, \quad \tau_h = \frac{1}{\alpha_h + \beta_h}$$

$$n_{close} \underset{\alpha_n}{\overset{\beta_n}{\rightleftarrows}} n_{open}$$

$$m_{close} \underset{\alpha_m}{\overset{\beta_m}{\rightleftarrows}} m_{open}$$

$$h_{close} \underset{\alpha_h}{\overset{\beta_h}{\rightleftarrows}} h_{open}$$



HH Membrane model

$$I_m = C_m \frac{dV}{dt} + g_K(V,t)(V - E_K) + g_{Na}(V,t)(V - E_{Na}) + g_L(V - E_L)$$

$$\alpha_n(V) = \frac{0.001(-V+10)}{e^{\frac{-V+10}{10}} - 1}$$

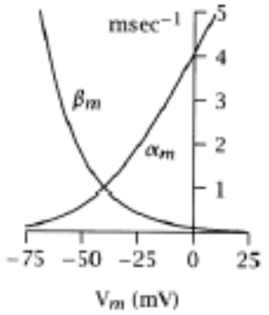
$$\beta_n(V) = 0.125 e^{\frac{-V}{80}}$$

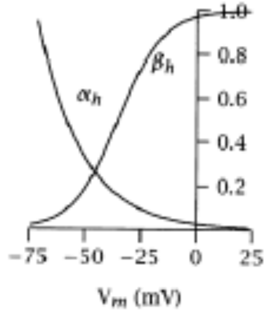
$$\alpha_m(V) = \frac{0.01(-V+25)}{e^{\frac{-V+25}{10}} - 1}$$

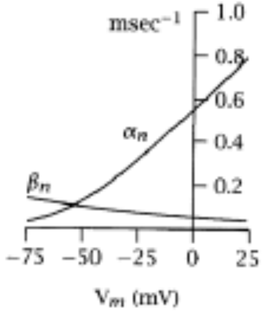
$$\beta_m(V) = 4 e^{\frac{-V}{18}}$$

$$\alpha_h(V) = 0.07 e^{\frac{-V}{20}}$$

$$\beta_h(V) = \frac{1}{e^{\frac{-V+30}{10}} + 1}$$







Membrane model evolvement

- HH model – Hodgkin and Huxley (1952)
- FH model – Frankenhaeuser and Huxley (1964)
- CRRSS model – Chiu, Ritchie, Rogert, Stagg and Sweeney (1979)
- SE model – Schwarz and Eikhof (1987)
- SRB model – Schwarz, Reid, and Bostock (1995)

Table 3.1 Expressions and Constants for Axon Membrane Models

	HH Model	FH Model	CRRSS Model	SE Model	SRB Model
α_m	$\frac{2.5-0.1V}{\exp(2.5-0.1V)-1}$	$\frac{0.36(V-22)}{1-\exp\left(\frac{22-V}{3}\right)}$	$\frac{97+0.363V}{1+\exp\left(\frac{31-V}{5.3}\right)}$	$\frac{1.87(V-25.41)}{1-\exp\left(\frac{25.41-V}{6.06}\right)}$	$\frac{4.6(V-65.6)}{1-\exp\left(\frac{-V+65.6}{10.3}\right)}$
β_m	$4.\exp\left(-\frac{V}{18}\right)$	$\frac{0.4(13-V)}{1-\exp\left(\frac{V-13}{20}\right)}$	$\frac{\alpha_m}{\exp\left(\frac{V-23.8}{4.17}\right)}$	$\frac{3.97(21-V)}{1-\exp\left(\frac{V-21}{9.41}\right)}$	$\frac{0.33(61.3-V)}{1-\exp\left(\frac{V-61.3}{9.16}\right)}$
α_n	$\frac{0.1-0.01V}{\exp(1-0.1V)-1}$	$\frac{0.02(V-35)}{1-\exp\left(\frac{35-V}{10}\right)}$		$\frac{0.13(V-35)}{1-\exp\left(\frac{35-V}{10}\right)}$	$\frac{0.0517(V+9.2)}{1-\exp\left(\frac{-V-9.2}{1.1}\right)}$
β_n	$0.125.\exp\left(-\frac{V}{80}\right)$	$\frac{0.05(10-V)}{1-\exp\left(\frac{V-10}{10}\right)}$		$\frac{0.32(10-V)}{1-\exp\left(\frac{V-10}{10}\right)}$	$\frac{0.092(8-V)}{1-\exp\left(\frac{V-8}{10.5}\right)}$