

C106B Discussion 9: Grasping

1 Introduction

Grasping objects is a major part of current robotic manipulation research. To approach this problem, we will discuss wrenches and their mathematical properties and then apply them to the idea of contact forces.

2 Wrenches

Last semester, we started our discussion of robotic arm movement talking about *kinematics*. This deals with the different positions and angles our body frame can potentially reach. The orientation of the **B** frame with respect to the **A** frame is given by the forward kinematic map:

$$g_{AB}(\theta_1) = e^{\hat{\xi}^1 \theta_1} g_{AB}(0)$$

We then discussed *kinetics*, which deal with velocities and accelerations. The relative velocity of a point given in the body frame for some angular velocity $\dot{\theta}$ is

$$v_{q_S} = \hat{V}_{AB}^s q_S, \quad V_{AB}^s = \xi' \dot{\theta} = \begin{bmatrix} v_{AB}^s \\ \omega_{AB}^s \end{bmatrix}$$
$$v_{q_B} = \hat{V}_{AB}^b q_B, \quad V_{AB}^b = \xi^\dagger \dot{\theta} = \begin{bmatrix} v_{AB}^b \\ \omega_{AB}^b \end{bmatrix}$$

where ξ' and ξ^\dagger are the current spatial and body twists.

Now, we move to dynamics for an arm - analyzing the relationship between *forces* applied on the body and its motion! A *wrench* follows the same kind of linear/angular form as twists:

$$\Gamma = \begin{bmatrix} f \\ \tau \end{bmatrix}$$

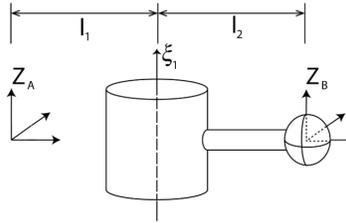
where f is a linear force component, and τ is a torque.

Unlike angular velocities, however, to compute the total torque on some joint, we use the *transpose* of the twist.

$$\tau = \xi'^T \Gamma^S$$
$$\tau = \xi^{\dagger T} \Gamma^B$$

The difference between the spatial and body wrench Γ s is the frame in which we are applying the wrench.

Problem 1: How does a wrench Γ^B applied on the **B** frame affect the torque at joint ξ_1 ?



3 Adjoints for Wrenches

Spatial and body velocities are related to one another using adjoints (which are invertible):

$$V_{AB}^S = Ad_{g_{AB}} V_{AB}^b$$

$$Ad_g = \begin{bmatrix} R & \hat{p}R \\ 0 & R \end{bmatrix}$$

Can we figure out a similar relationship for wrenches?

It turns out we can!

$$\Gamma^S = Ad_{g_{AB}}^T \Gamma^B$$

Problem 2: The work of a force is calculated by $W = F \cdot d$. Two wrenches are equivalent if they generate the same amount of work. Use this concept to prove the adjoint relationship for wrenches.

4 Jacobians for Wrenches

Last semester, we used the spatial and body Jacobians to transform individual joint velocities to the end-effector velocity and vice-versa. You also used this concept in Project 1 to generate jointspace trajectory commands. Recall that

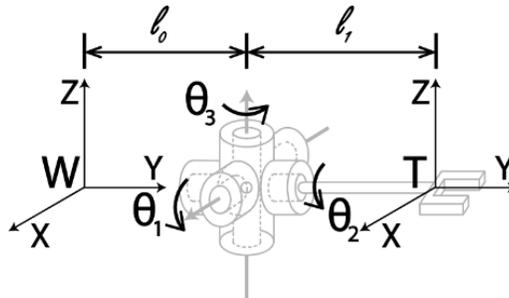
$$\begin{aligned} V^S &= J^S(\theta)\dot{\theta} \\ V^B &= J^B(\theta)\dot{\theta} \end{aligned}$$

where $\dot{\theta}$ is a vector of individual joint velocities. The Jacobian itself is a composition of the individual joint twists in their current configuration.

To compute the torques on each joint based on a wrench applied in the spatial or body frame, we can use the Jacobian as well:

$$\begin{bmatrix} \tau_1 \\ \vdots \\ \tau_n \end{bmatrix} = (J^S)^T \Gamma^S = (J^B)^T \Gamma^B$$

Problem 3: Compute the joint torques if we apply a force on the body frame.



5 Grasp map

When going into robotic hands, we want to actually grab objects. One *contact* is defined by

$$F_{c_i} = B_{c_i} f_{c_i}$$

Where B is the contact basis, or the directions in which the contact can apply force, and f is a vector in that basis (the actual forces being applied). F is the 6x1 wrench which the contact applies. In our case, we use a soft contact model, which has both lateral and torsional friction components, so the basis is

$$B_{c_i} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

However, in the real world, friction is not infinite. For the contact to resist a wrench without slipping, the contact vector must lie within the *friction cone*, which is defined

$$FC_{c_i} = \{f \in \mathbb{R}^4 : \sqrt{f_1^2 + f_2^2} \leq \mu f_3, f_3 > 0, |f_4| \leq \gamma f_3\}$$

f_3 is the amount of normal force being applied, f_1 and f_2 are the forces in the other two perpendicular directions, and f_4 is a torque. The friction cone therefore tells us the forces that can be applied onto an object that would be resisted by this contact.

However, we want the wrenches that a contact point can resist in the world frame, not the contact frame. So we use the adjoint to transform the contact basis:

$$G_i := \begin{bmatrix} R_{oc_i} & 0 \\ \widehat{p}_{oc_i} R_{oc_i} & R_{oc_i} \end{bmatrix} B_{c_i} = Ad_{g_{co_i}}^T B_{c_i}$$

A grasp is a *set* of contacts (maybe multiple fingers in a hand or the two sides of a Sawyer gripper), so we define the wrenches (in the world frame) a grasp can resist as:

$$F_o = G_1 f_{c_1} + \dots + G_k f_{c_k} = \begin{bmatrix} G_1 & \dots & G_k \end{bmatrix} \begin{bmatrix} f_{c_1} \\ \vdots \\ f_{c_k} \end{bmatrix} = Gf$$

The resulting compound matrix G above is called the *grasp map*, summing up multiple forces.

6 Force closure

A grasp is in *force closure* when finger forces lying in the friction cones span the space of object wrenches

$$G(FC) = \mathbb{R}^6$$

Essentially, this means that any external wrench applied to the object can be countered by the sum of contact forces (provided the contact forces are high enough).

For a two-contact soft-fingered grasp, we also have the following theorem which makes it very easy to check when a grasp is in force closure. This is theorem 5.7 from MLS.

Theorem. *A spatial grasp with two soft-finger contacts is force-closure if and only if the line connecting the contact point lies inside both friction cones.*

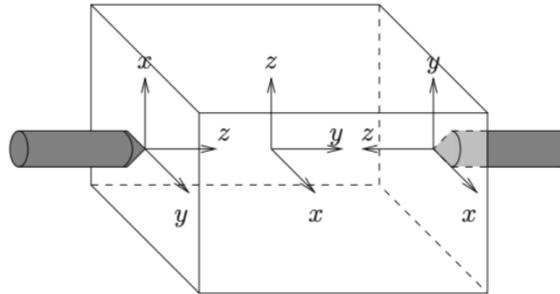


Figure 1: Two finger grasp.

6.1 Discretizing the Friction Cone

Checking that $f \in FC$ can be difficult. Often when evaluating grasps, we will write down an optimization problem that has $f \in FC$ as a constraint.

$$FC_{c_i} = \begin{cases} \sqrt{f_1^2 + f_2^2} \leq \mu f_3 \\ f_3 > 0 \\ |f_4| \leq \gamma f_3 \end{cases}$$

We can approximate the (conical) friction cone as a pyramid with n vertices. The level sets of the friction cone are circles, but the level sets for this convex approximation are n sided polygons circumscribed by the circle. Thus, the interior of this convexified friction cone is a conservative approximation of the friction cone itself.

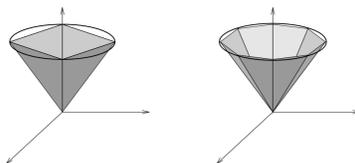


Figure 2: Approximations of the friction cone. From section 5.3 of MLS.

Any point in the interior of this pyramid can be described as a sum of

$$f = \alpha_0 f_0 + \sum_{i=1}^n \alpha_i f_i = F\alpha$$

where f_i are the edges of the pyramid and f_0 a straight line in z , and the weights α are all non-negative. Here, we can write a composite matrix F (different from the F above!) with the f_i vectors as its columns. This lets us more easily characterize any f in the friction cone. We make the approximation that $f \in FC$ if and only if there exists a non-negative vector α such that $f = F\alpha$.

With this approximation, the condition that $f \in FC$ is equivalent to the pair of linear constraints $\{f = F\alpha, \alpha \geq 0\}$ (where this inequality is understood to be element-wise).

Problem 4:

Let w be a given wrench. Let a two-contact grasp be given to you with contact grasp maps G_1 and G_2 . We wish to find the input force $f \in FC$ with the smallest norm that can resist the wrench w applied at the center of mass of the object being grasped. Using the polyhedral approximation of the friction cone, write this as a quadratic program.

Problem 5:

Consider the box grasped by 2 soft-finger contacts shown in the figure above. Find the grasp map. Assume the object is a cube of side-length 2.