CS 170: Algorithms

Logistics:
Course website: https://cs170.org/ (lots of information)

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Course communications: Edstem

CS 170: Algorithms

Logistics:

Course website: https://cs170.org/ (lots of information)

Course communications: Edstem

Weekly homeworks + 2 midterms + 1 final exam.

CS 170: Algorithms

Logistics:
Course website: https://cs170.org/ (lots of information)

Course communications: Edstem

Weekly homeworks + 2 midterms + 1 final exam.

Midterm 1: M 2/26/24 from 7pm-9pm.
Midterm 2: Tu 4/2/24 from 7pm-9pm

CS 170: Algorithms

Logistics:
Course website: https://cs170.org/ (lots of information)

Course communications: Edstem

Weekly homeworks + 2 midterms + 1 final exam.

Midterm 1: M 2/26/24 from 7pm-9pm.
Midterm 2: Tu 4/2/24 from 7pm-9pm

Instructors:

CS 170: Algorithms

Logistics:
Course website: https://cs170.org/ (lots of information)

Course communications: Edstem

Weekly homeworks + 2 midterms + 1 final exam.

Midterm 1: M 2/26/24 from 7pm-9pm.
Midterm 2: Tu 4/2/24 from 7pm-9pm

Instructors:
Christian Borgs

CS 170: Algorithms

Logistics:
Course website: https://cs170.org/ (lots of information)

Course communications: Edstem

Weekly homeworks + 2 midterms + 1 final exam.

Midterm 1: M 2/26/24 from 7pm-9pm.
Midterm 2: Tu 4/2/24 from 7pm-9pm

Instructors:
Christian Borgs
Prasad Raghavendra

CS 170: Algorithms

Logistics:
Course website: https://cs170.org/ (lots of information)

Course communications: Edstem

Weekly homeworks + 2 midterms + 1 final exam.

Midterm 1: M 2/26/24 from 7pm-9pm.
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Instructors:
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Prasad Raghavendra

CS 170: Algorithms

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Course communications: Edstem

Weekly homeworks + 2 midterms + 1 final exam.

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Instructors:
Christian Borgs
Prasad Raghavendra

Today: Satish Rao

A puzzle.

Does a list have a cyle?

A puzzle.

Does a list have a cyle?

Access to list is a pointer to the “first element.”

A puzzle.

Does a list have a cyle?

Access to list is a pointer to the “first element.”

Mark first node.

A puzzle.

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Mark first node.
While next cell not marked, go to next cell.

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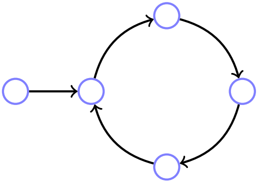

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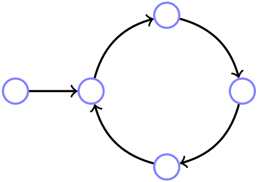

First node not in cycle!

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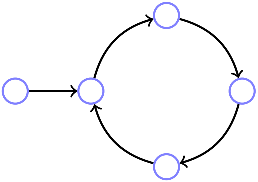

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Answer is no.

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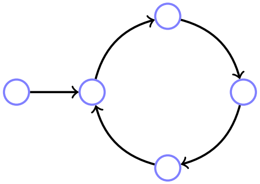

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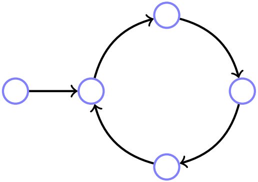

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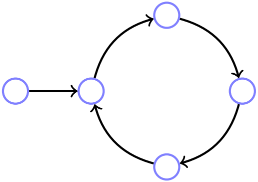

First node not in cycle!

Answer is no. “Oracle” gave us example.

Problem: starting point is not on cycle?
Construct example.

Does a list have a cyle?

Does a list have a cyle?

Two ptrs:

Does a list have a cyle?

Two ptrs: Step: advance ptr 1 twice,

Does a list have a cyle?

Two ptrs: Step: advance ptr 1 twice, advance ptr 2 once.

Does a list have a cyle?

Two ptrs: Step: advance ptr 1 twice, advance ptr 2 once.
If ever at the same place, report cycle.

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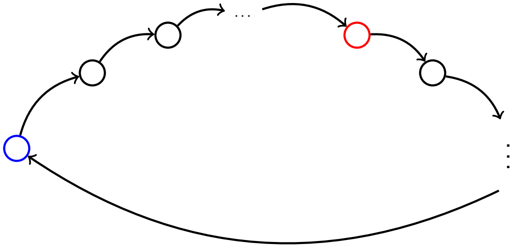

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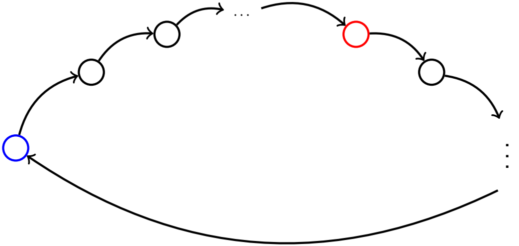

Correctness:

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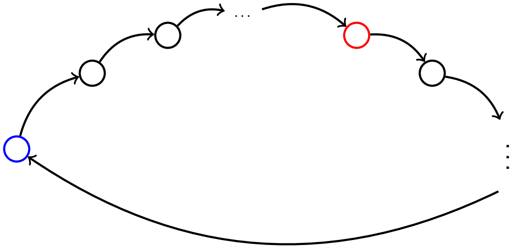
Two ptrs: Step: advance ptr 1 twice, advance ptr 2 once.
If ever at the same place, report cycle.



Correctness:
If no cycle, slow pointer never catches fast one.

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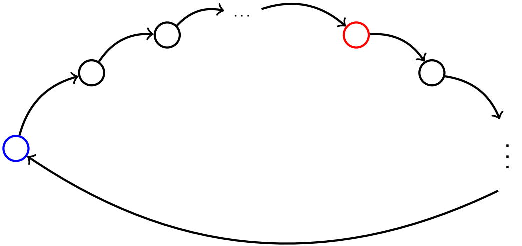

Correctness:
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If cycle, both pointers will enter cycle at some time.

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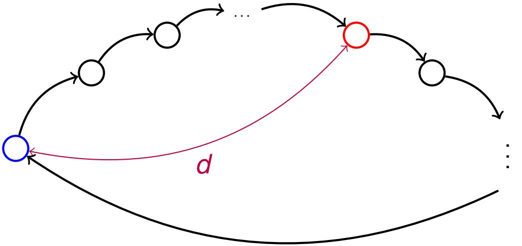

Correctness:
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d - distance from fast ptr to slow ptr.

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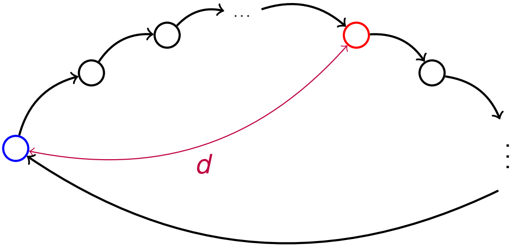

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If no cycle, slow pointer never catches fast one.
If cycle, both pointers will enter cycle at some time.
d - distance from fast ptr to slow ptr.

d decreases every step.

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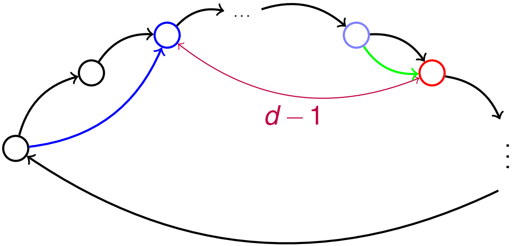

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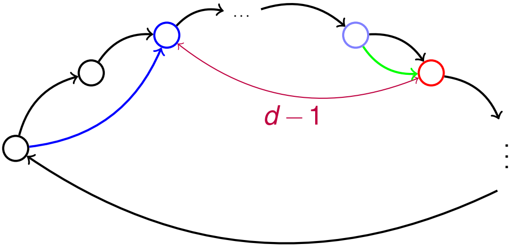

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Runtime: n steps to cycle

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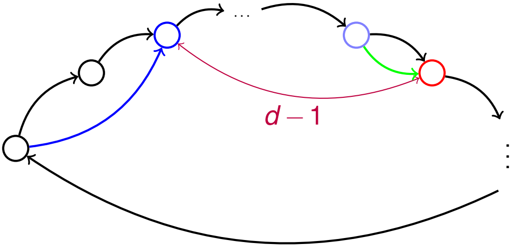

Correctness:
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d decreases every step.

Runtime: n steps to cycle n steps to catch up.

Does a list have a cyle?

Two ptrs: Step: advance ptr 1 twice, advance ptr 2 once.
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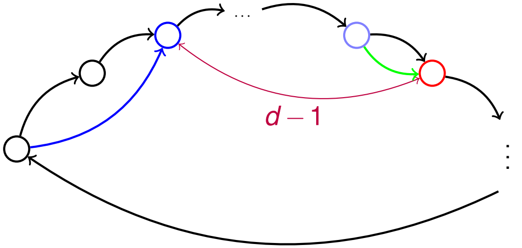

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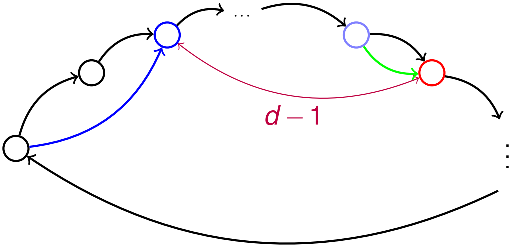

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Additional storage: two pointers.

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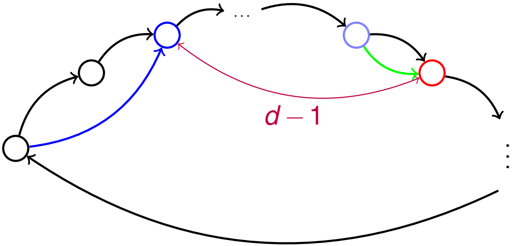

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d decreases every step.

Runtime: n steps to cycle n steps to catch up. $O(n)$
Additional storage: two pointers. $O(1)$.

Puzzles ..

Solutions

Puzzles ..

Solutions

..are Algorithms...

Puzzles ..

Solutions

..are Algorithms...

which...

Puzzles ..

Solutions

..are Algorithms...

which...

..are correct...

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..are correct...and (in this class) efficient.

Puzzles ..

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Is this a useful process?

Algorithms for the Human Genome Project

Reconstruct DNA...

Algorithms for the Human Genome Project

Reconstruct DNA...

ACTGAAACTGAGTAGATA....

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Read first, then next, then next, ...3.1 billion times...
.. slow... error prone...

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Parallel sequencing yields chunks of overlapping DNA.

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        TGAGT
          AGTAG
             AGATA
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Page Rank.

Problem: What is good on the web?

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Website that pays search engine most?

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Random Surfer Model (Brin-Page):

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Issues: make a bunch of webpages that point to each other.

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New Model for user: google.

Algorithms...

Driving Directions

Algorithms...

Driving Directions
Airline Scheduling

Algorithms...

Driving Directions
Airline Scheduling
Compiling

Algorithms...

Driving Directions
Airline Scheduling
Compiling
Compression

Algorithms...

Driving Directions
Airline Scheduling
Compiling
Compression
Cryptography

Algorithms...

Driving Directions
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Compiling
Compression
Cryptography
Optimization

Algorithms...

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Compression
Cryptography
Optimization
Search

Algorithms...

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Cryptography
Optimization
Search
Targeted Advertising

Algorithms...

Driving Directions
Airline Scheduling
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.
.

Calculating: 300 BC through Middle Ages in Europe.

I – one

Calculating: 300 BC through Middle Ages in Europe.

I – one
V – five

Calculating: 300 BC through Middle Ages in Europe.

I – one
V – five
X – ten

Calculating: 300 BC through Middle Ages in Europe.

I – one
V – five
X – ten
C – one hundred

Calculating: 300 BC through Middle Ages in Europe.

I – one
V – five
X – ten
C – one hundred
D – five hundred

Calculating: 300 BC through Middle Ages in Europe.

I – one
V – five
X – ten
C – one hundred
D – five hundred
M – a thousand

Calculating: 300 BC through Middle Ages in Europe.

I – one
V – five
X – ten
C – one hundred
D – five hundred
M – a thousand

Calculating: 300 BC through Middle Ages in Europe.

I – one
V – five
X – ten
C – one hundred
D – five hundred
M – a thousand

VIII – eight

Calculating: 300 BC through Middle Ages in Europe.

I – one
V – five
X – ten
C – one hundred
D – five hundred
M – a thousand

VIII – eight

DCLXXVI – five hundred plus a hundred plus fifty plus ten plus ten..

Calculating: 300 BC through Middle Ages in Europe.

I – one
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MCDLXVIII – one thousand five hundred minus one hundred ....

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Add them?

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Add them?

1448 + 676 =

Calculating: 300 BC through Middle Ages in Europe.

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Add them?

1448 + 676 = 2024

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MCDLXVIII – one thousand five hundred minus one hundred ....

Add them?

1448 + 676 = 2024

676 years since the Gutenberg printing press.

Calculating: 300 BC through Middle Ages in Europe.

I – one
V – five
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From India, via Al Khwarizmi.

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The input representation for modern computers and communication.

Writing to propagating..

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Italian mathematician (1170-1250) who traveled to learn the Hindu-Arab math.

Place value.

I love place value!!

Place value.

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Democratizes arithmetic.

Place value.

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Democratizes arithmetic. Money.

Place value.

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Democratizes arithmetic. Money. Helps end feudal system?

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N in decimal takes $\lceil \log_{10} N \rceil$ digits.

Fibonacci numbers.

$F_0 = 0$, $F_1 = 1$.

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$T(n) \geq F_n$

Fibonacci algorithm and numbers!

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Can we do better?

Better Algorithm.

Better Algorithm.

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Oops:
doubling the size more than doubled the runtime!

From demo: Size matters.

How many bits in the representation of F_n?

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How long does Fib take?

n additions.

At most $O(n^2)$.

Demo: polynomial.

Doubling size, made fast fib grow by factor of roughly four.

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Example: cn^2 runtime.

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From: $2^{ab} = (2^a)^b = (2^b)^a$.

Asymptotic Analysis.

Used $O(n)$ for number of additions, rather than $n-2$.

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$2^{.694n}$ versus $O(n^2)$.

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  Calculation: $c(2n)^2 = 4 \times cn^2$

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Refreshing Asymptotic Notation.

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Upper bound.
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Formally, for positive functions g, f from integers to reals,
$g(n) = O(f(n))$, if there is a constant c where $g(n) \leq cf(n)$.

More asymptotic notation.

Ω notation.

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$g(n) = \Theta(f(n))$ if $g(n) = O(f(n))$ and $g(n) = \Omega(f(n))$.

Al Khwarizmi: Arithmetic.

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Addition: $O(n)$

Al Khwarizmi: Arithmetic.

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Place value.

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Place value. Algorithm: add places.

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Al Khwarizmi: Arithmetic.

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$$
\begin{array}{cccccccccc}
 & & & & & 1 & 1 & 1 & 1 & \\
 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\
+ & 9 & 2 & 1 & 2 & 3 & 7 & 6 & 9 & 1 \\
\hline
 & & & & & & 4 & 4 & 8 & 0
\end{array}
$$

Al Khwarizmi: Arithmetic.

Addition: $O(n)$

Place value. Algorithm: add places.

$$
\begin{array}{cccccccccc}
 & & & & 0 & 1 & 1 & 1 & 1 & \\
 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\
+ & 9 & 2 & 1 & 2 & 3 & 7 & 6 & 9 & 1 \\
\hline
 & & & & & 9 & 4 & 4 & 8 & 0
\end{array}
$$

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		0	0	0	1	1	1	1	
	1	2	3	4	5	6	7	8	9
+	9	2	1	2	3	7	6	9	1
			4	6	9	4	4	8	0

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Correctness:

Al Khwarizmi: Arithmetic.

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1	0	0	0	0	1	1	1	1	
	1	2	3	4	5	6	7	8	9
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Correctness:

See how many ones, if more than 10, add to 10's.

Al Khwarizmi: Arithmetic.

Addition: $O(n)$

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\begin{array}{cccccccccc}
1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & \\
 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\
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$$

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And so on.

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Can we do better?

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1	0	0	0	0	1	1	1	1	
	1	2	3	4	5	6	7	8	9
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Need to look at the numbers to add them...

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And so on.

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Need to look at the numbers to add them... must be optimal.

More Al Khwarizmi's: algorithms.

Addition: $O(n)$

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Addition: $O(n)$

Multiplication:

$$
\begin{array}{rrrrrrrrrr}
 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\
\times & 9 & 2 & 1 & 2 & 3 & 7 & 6 & 9 & 1 \\
\hline
\end{array}
$$

More Al Khwarizmi's: algorithms.

Addition: $O(n)$

Multiplication:

$$
\begin{array}{rccccccccc}
 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\
\times & 9 & 2 & 1 & 2 & 3 & 7 & 6 & 9 & 1 \\
\hline
 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9
\end{array}
$$

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 & & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\
 & \times & 9 & 2 & 1 & 2 & 3 & 7 & 6 & 9 & 1 \\
\hline
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 & & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\
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. & . & . & . & . & . & . & . & . & . & .
\end{array}$$

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 & & & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\
 & 9 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 1 & \\
 & . & . & . & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & . & . & . & .
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$$

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 & & 9 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 1 & \\
 & & . & . & . & . & . & . & . & . & . & . & . \\
 & . & . & . & . & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & . & . & . & . & .
\end{array}
\quad \updownarrow n
$$

More Al Khwarizmi's: algorithms.

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 & & & & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\
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 & . & . & . & . & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & . & . & . & . & .
\end{array}
$$

n

Time: $O(n^2)$

Multiplication

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Is the best possible?

Multiplication

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Is the best possible?

Every digit in x must multiply every digit in y at least once!

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(a) Yes.

(b) No.

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(b) No.

No.

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What does python do?

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Multiplication: $O(n^2)$.

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What does python do? Let's see.

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What does python do? Let's see.

Runtime:
 2 times as large increases by a factor of 3

Multiplication

Multiplication: $O(n^2)$.

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What does python do? Let's see.

Runtime:
2 times as large increases by a factor of 3
Note: $2^{\log_2 3} = 3$.

Multiplication

Multiplication: $O(n^2)$.

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Every digit in x must multiply every digit in y at least once!
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2 times as large increases by a factor of 3
Note: $2^{\log_2 3} = 3$.

Runtime: $O(n^{\log_2 3})$.

Multiplication

Multiplication: $O(n^2)$.

Is the best possible?

Every digit in x must multiply every digit in y at least once!
$\Theta(n^2)$ such pairs.

Is this the best possible?

(a) Yes.

(b) No.

No. We can do better!
What ?!?!
Really!

What does python do? Let's see.

Runtime:
2 times as large increases by a factor of 3
Note: $2^{\log_2 3} = 3$.

Runtime: $O(n^{\log_2 3})$.
How?

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How? Next time (read ahead!!!)

Quick Thoughts.

Big (historic) idea: representation as digits or bits.

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Asymptotic analysis.