CS170: Lecture 2

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Last Time: Place value is democratizing!

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  Like the printing press!

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Last Time: Place value is democratizing!
Like the printing press!
Reading, writing, arithmetic!

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Input size/representation really matters!

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Today: Chapter 2.

CS170: Lecture 2

Last Time: Place value is democratizing!
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Today: Chapter 2.
Divide and Conquer

CS170: Lecture 2

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Today: Chapter 2.
 Divide and Conquer $\equiv$ Recursive.

CS170: Lecture 2

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Today: Chapter 2.
 Divide and Conquer $\equiv$ Recursive.

Lecture in one minute!

Integer Multiplication: Gauss plus recursion is magic!

$O(n^2) \rightarrow O(n^{\log_2 3}) \approx O(n^{1.58..})$

Lecture in one minute!

Integer Multiplication: Gauss plus recursion is magic!

$O(n^2) \rightarrow O(n^{\log_2 3}) \approx O(n^{1.58..})$

Double size, time grows by a factor of 3.

Master's theorem: understand the recursion tree!

$T(n) = aT(\frac{n}{b}) + f(n)$.

Branching by a

diminishing by b

working by $O(f(n))$.

Leaves: $n^{\log_b a}$, Work: $\sum_i a^i f(\frac{n}{b^i})$.

Lecture in one minute!

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Recursive (Divide and Conquer) Matrix Multiplication:

8 subroutine calls of size $n/2 \times n/2$

$\rightarrow O(n^3)$.

Strassen:

7 subroutine calls of size $n/2 \times n/2$

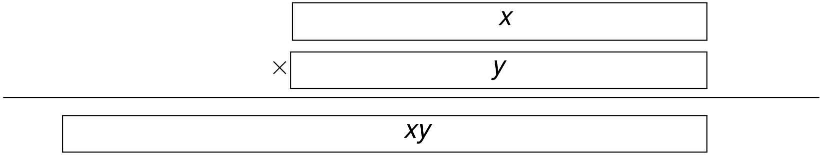
$\rightarrow O(n^{\log_2 7}) \approx O(n^{2.8})$.

Chapter 2.

Divide and conquer.

Definition of Multiplication.

n-bit numbers: x, y.



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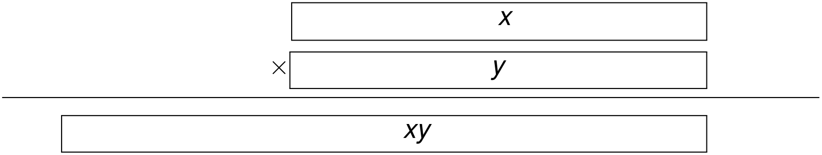
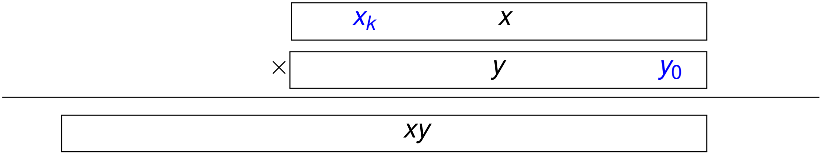


kth "place" of xy:

Definition of Multiplication.

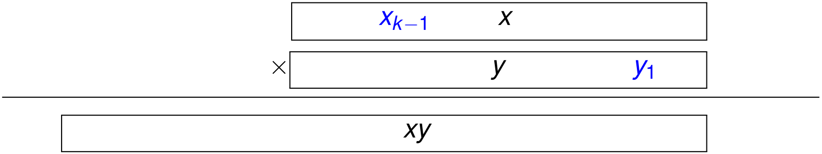
n-bit numbers: x, y.



kth "place" of xy: coefficient of 2^k:

Definition of Multiplication.

n-bit numbers: x, y.



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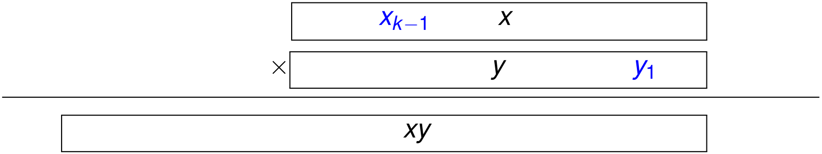

kth "place" of xy: coefficient of 2^k:

$$a_k = \sum_{i \leq k} x_i y_{k-i}.$$

Definition of Multiplication.

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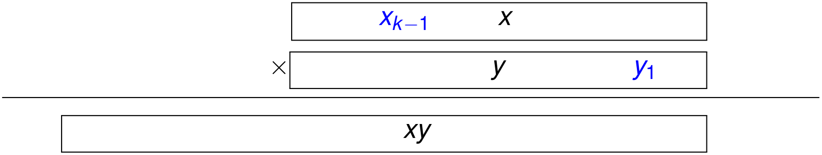


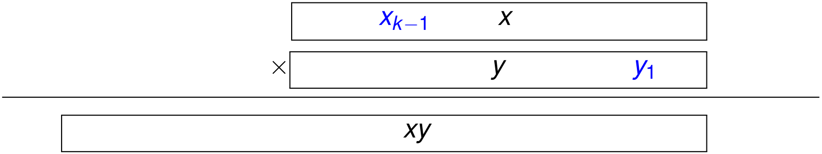
kth "place" of xy: coefficient of 2^k:

$$a_k = \sum_{i \le k} x_i y_{k-i}.$$

$x * y = \sum_{k=0}^{2n} 2^k a_k$.

Definition of Multiplication.

n-bit numbers: x, y.



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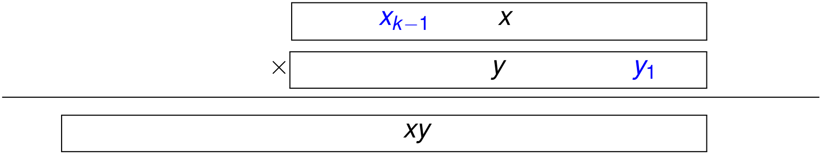

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$x * y = \sum_{k=0}^{2n} 2^k a_k$.

Number of "basic operations":

$$\sum_{k \le 2n} \min(k, 2n-k) = \Theta(n^2).$$

Recursive Algorithm for Multiplication.

Two n-bit numbers: x, y.

Recursive Algorithm for Multiplication.

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$x = $ | x_L | x_R |

Recursive Algorithm for Multiplication.

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$$x = \boxed{\quad x_L \quad}\boxed{\quad x_R \quad} = 2^{n/2}x_L + x_R$$

Recursive Algorithm for Multiplication.

Two n-bit numbers: x, y.

x	$=$	x_L	x_R	$= 2^{n/2}x_L + x_R$
y	$=$	y_L	y_R	

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Multiplying out

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Multiplying out

$$x \times y$$

Recursive Algorithm for Multiplication.

Two n-bit numbers: x, y.

$x =$	x_L	x_R
$y =$	y_L	y_R

$$x = 2^{n/2}x_L + x_R$$
$$y = 2^{n/2}y_L + y_R$$

Multiplying out

$$x \times y = (2^{n/2}x_L + x_R)(2^{n/2}y_L + y_R)$$

Recursive Algorithm for Multiplication.

Two n-bit numbers: x, y.

x	$=$	x_L	x_R	$= 2^{n/2}x_L + x_R$
y	$=$	y_L	y_R	$= 2^{n/2}y_L + y_R$

Multiplying out

$$\begin{aligned} x \times y &= (2^{n/2}x_L + x_R)(2^{n/2}y_L + y_R) \\ &= 2^n x_L y_L + 2^{n/2}(x_L y_R + x_R y_L) + x_R y_R \end{aligned}$$

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Two n-bit numbers: x, y.

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Four $n/2$-bit multiplications: $x_L y_L$, $x_L y_R$, $x_R y_L$, $x_R y_R$.

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Four $n/2$-bit multiplications: $x_L y_L$, $x_L y_R$, $x_R y_L$, $x_R y_R$.
Recurrence:

$$T(n) = 4T(\frac{n}{2}) + O(n)$$

Recurrence for recursive algorithm.

Recurrence:

$$T(n) = 4T(\frac{n}{2}) + \Theta(n)$$

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$T(n)$ is

(A) $\Theta(n)$.

(B) $\Theta(n^2)$.

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Idea: Think about recursion tree.
A degree 4 tree of depth $\log_2 n$.

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$4^{\log_2 n}$

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$4^{\log_2 n} = (2^2)^{\log_2 n}$

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One for each pair of digits!

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Really?

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Really? Unfolded recursion in my head?!?!

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How did I really obtain bound?

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TBH,unfolded recurrence in head. Don't remember formulas.

Demo

As number of bits double:

Demo

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Demo

As number of bits double:

Elementary School Multiply:

Demo

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$O(n^2)$
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Runtime: $T = cn^2 \rightarrow T' = c(2n)^2 = 4(cn^2) = 4T$

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Python multiply:

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Runtime: $T \rightarrow 3T$.

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Asymptotics: $T = cn^w \to c((2n)^w) = T' = 3T = 3(cn^w)$.

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Much better than grade school.

Multiply Complex Numbers

$$(3+2\,\mathbf{i})(4+5\,\mathbf{i}) = 12+(15+8)\,\mathbf{i}+10\,\mathbf{i}^2$$

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$$(12-10)+22\,\mathbf{i} = 2+22\,\mathbf{i}.$$

Multiply Complex Numbers

$(3+2\,\mathbf{i})(4+5\,\mathbf{i}) = 12+(15+8)\,\mathbf{i}+10\,\mathbf{i}^2$

Recall, $\mathbf{i}^2 = -1$, so simplifying

$(12-10)+22\,\mathbf{i} = 2+22\,\mathbf{i}$.

What about $(32765+219898\,\mathbf{i})(413764+511110\,\mathbf{i})$?

Gauss's trick.

$$(a + b\,\mathbf{i})(c + d\,\mathbf{i})$$

Gauss's trick.

$$(a+b\,\mathbf{i})(c+d\,\mathbf{i}) = (ac-bd)+(ad+bc)\,\mathbf{i}.$$

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Only three multiplications.

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Only three multiplications. An extra addition though!
Which is harder of multiplication or addition?

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Which is harder of multiplication or addition?
Multiplication!

Faster Algorithm for Multiplication.

Two n-bit numbers: x, y.

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$$x = 2^{n/2} x_L + x_R$$

Faster Algorithm for Multiplication.

Two n-bit numbers: x, y.

$$x = 2^{n/2} x_L + x_R \quad ; \quad y = 2^{n/2} y_L + y_R$$

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Need 3 terms: $x_L y_L$, $x_L y_R + x_R y_L$, $x_R y_R$.

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Used four $\frac{n}{2}$-bit multiplications: $x_L y_L$, $x_L y_R$, $x_R y_L$, $x_R y_R$.

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Can you compute three terms with 3 multiplications?

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Can you compute three terms with 3 multiplications?

(A) Yes.

(B) No

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(A) Yes.

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(A) Yes.

Three multiplications and faster algorithm.

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Need 3 terms: $x_L y_L$, $x_L y_R + x_R y_L$, $x_R y_R$.

Compute

$$P_1 = (x_L + x_R)(y_L + y_R)$$

Three multiplications and faster algorithm.

Two n-bit numbers: x, y.

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$$P_1 = (x_L + x_R)(y_L + y_R) = x_L y_L + x_L y_R + x_R y_L + x_R y_R.$$

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Compute

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Two more: $P_2 = x_L y_L$, $P_3 = x_R y_R$.

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3 multiplications!

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$$T(n) = 3T(\frac{n}{2}) + \Theta(n)$$

Three multiplications and faster algorithm.

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Technically: $\frac{n}{2} + 1$ bit multiplication.

Three multiplications and faster algorithm.

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3 multiplications!

$$T(n) = 3T(\frac{n}{2}) + \Theta(n)$$

Technically: $\frac{n}{2} + 1$ bit multiplication. Don't worry.

Analysis of runtime.

Recurrence for “fast algorithm”.

$$T(n) = 3T(\frac{n}{2}) + \Theta(n)$$

Analysis of runtime.

Recurrence for “fast algorithm”.

$$T(n) = 3T(\frac{n}{2}) + \Theta(n)$$

Runtime is

(A) $\Theta(n)$

(B) $\Theta(n^2)$

Analysis of runtime.

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$$T(n) = 3T(\frac{n}{2}) + \Theta(n)$$

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(C) $\Theta(n^{\log_2 3})$

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$3^{\log_2 n}$

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(C) Idea: number of base cases is $n^{\log_2 3}$.

$3^{\log_2 n} = (2^{\log_2 3})^{\log_2 n}$

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So multiplication algorithm with ..

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So multiplication algorithm with ..

$$T(n) = 3T(\frac{n}{2}) + \Theta(n) = \Theta(n^{\log_2 3}) = \Theta(n^{1.58\ldots})$$

Analysis of runtime.

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$$T(n) = 3T(\frac{n}{2}) + \Theta(n) = \Theta(n^{\log_2 3}) = \Theta(n^{1.58...})!$$

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So multiplication algorithm with ..

$$T(n) = 3T(\frac{n}{2}) + \Theta(n) = \Theta(n^{\log_2 3}) = \Theta(n^{1.58...})!!!!$$

But: all digits have to multiply each other!

Analysis of runtime.

Recurrence for "fast algorithm".

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Runtime is

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$$T(n) = 3T(\frac{n}{2}) + \Theta(n) = \Theta(n^{\log_2 3}) = \Theta(n^{1.58...})!!!!$$

But: all digits have to multiply each other!
They do!

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They do! $(a+b)(c+d) = ac + ac + bc + bd$

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But: all digits have to multiply each other!
They do! $(a+b)(c+d) = ac + ac + bc + bd$
4 products from one multiplication!

Logarithms reminder.

Exponents Quiz: $(a^b)^c = (a^c)^b$?

Logarithms reminder.

Exponents Quiz: $(a^b)^c = (a^c)^b$?

Yes? No?

Logarithms reminder.

Exponents Quiz: $(a^b)^c = (a^c)^b$?

Yes? No?

Yes. $(a^b)^c$

Logarithms reminder.

Exponents Quiz: $(a^b)^c = (a^c)^b$?

Yes? No?

Yes. $(a^b)^c = a^{bc}$

Logarithms reminder.

Exponents Quiz: $(a^b)^c = (a^c)^b$?

Yes? No?

Yes. $(a^b)^c = a^{bc} = a^{cb}$

Logarithms reminder.

Exponents Quiz: $(a^b)^c = (a^c)^b$?

Yes? No?

Yes. $(a^b)^c = a^{bc} = a^{cb} = (a^c)^b$.

Logarithms reminder.

Exponents Quiz: $(a^b)^c = (a^c)^b$?

Yes? No?

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Definition of $\log$:

Logarithms reminder.

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Definition of log: $a = b^{\log_b a}$

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Logarithm Quiz: $a^{\log_b n} = n^{\log_b a}$?

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$$a^{\log_b n}$$

Logarithms reminder.

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Logarithm Quiz: $a^{\log_b n} = n^{\log_b a}$?

Yes!

$$a^{\log_b n} = (b^{\log_b a})^{\log_b n} = (b^{\log_b n})^{\log_b a} = n^{\log_b a}$$

Solving recurrences.

$$T(n) = 4T(\frac{n}{2}) + cn; \qquad T(1) = c$$

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$$T(n) = 4T(\frac{n}{2}) + cn; \qquad T(1) = c$$

Recursion Tree	# probs	sz	time/prob	time/level
$T(n)$	1	n	cn	cn
$T(\frac{n}{2})$ $T(\frac{n}{2})$ $T(\frac{n}{2})$ $T(\frac{n}{2})$	4	$\frac{n}{2}$	$c(\frac{n}{2})$	$2cn$
$T(\frac{n}{4})\cdots$ $T(\frac{n}{4})$ $T(\frac{n}{4})$ $\cdots T(\frac{n}{4})$	4^2	$\frac{n}{4}$	$c(\frac{n}{4})$	$4cn$
	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\vdots$	4^i	$\frac{n}{2^i}$	$c(\frac{n}{2^i})$	$2^i cn$

Solving recurrences.

$$T(n) = 4T(\frac{n}{2}) + cn; \qquad T(1) = c$$

Recursion Tree	# probs	sz	time/prob	time/level
$T(n)$	1	n	cn	cn
$\swarrow \; \swarrow \quad \searrow \; \searrow$				
$T(\frac{n}{2}) \quad T(\frac{n}{2}) \quad T(\frac{n}{2}) \quad T(\frac{n}{2})$	4	$\frac{n}{2}$	$c(\frac{n}{2})$	$2cn$
$\swarrow \cdots \searrow \quad \cdots \quad \swarrow \cdots \searrow$				
$T(\frac{n}{4}) \cdots \; T(\frac{n}{4}) \quad T(\frac{n}{4}) \; \cdots T(\frac{n}{4})$	4^2	$\frac{n}{4}$	$c(\frac{n}{4})$	$4cn$
$\swarrow \; \searrow \; \swarrow \; \searrow \; \swarrow \; \searrow \; \swarrow \; \searrow$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots$	4^i	$\frac{n}{2^i}$	$c(\frac{n}{2^i})$	$2^i cn$

$\frac{n}{2^i} = 1$ when $i = \log_2 n$

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Recursion Tree	# probs	sz	time/prob	time/level
$T(n)$	1	n	cn	cn
$\swarrow \; \swarrow \quad \searrow \; \searrow$				
$T(\frac{n}{2}) \quad T(\frac{n}{2}) \quad T(\frac{n}{2}) \quad T(\frac{n}{2})$	4	$\frac{n}{2}$	$c(\frac{n}{2})$	$2cn$
$\swarrow \cdots \searrow \quad \cdots \quad \swarrow \cdots \searrow$				
$T(\frac{n}{4}) \cdots \; T(\frac{n}{4}) \quad T(\frac{n}{4}) \; \cdots T(\frac{n}{4})$	4^2	$\frac{n}{4}$	$c(\frac{n}{4})$	$4cn$
$\swarrow \; \searrow \; \swarrow \; \searrow \; \swarrow \; \searrow \; \swarrow \; \searrow$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots$	4^i	$\frac{n}{2^i}$	$c(\frac{n}{2^i})$	$2^i cn$

$\frac{n}{2^i} = 1$ when $i = \log_2 n \implies$ Depth: $d = \log_2 n$.

Solving recurrences.

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Recursion Tree	# probs	sz	time/prob	time/level
$T(n)$	1	n	cn	cn
$\swarrow \; \swarrow \quad \searrow \; \searrow$				
$T(\frac{n}{2}) \quad T(\frac{n}{2}) \quad T(\frac{n}{2}) \quad T(\frac{n}{2})$	4	$\frac{n}{2}$	$c(\frac{n}{2})$	$2cn$
$\swarrow \cdots \searrow \quad \cdots \quad \swarrow \cdots \searrow$				
$T(\frac{n}{4}) \cdots \quad T(\frac{n}{4}) \quad T(\frac{n}{4}) \quad \cdots T(\frac{n}{4})$	4^2	$\frac{n}{4}$	$c(\frac{n}{4})$	$4cn$
$\swarrow \; \searrow \; \swarrow \; \searrow \; \swarrow \; \searrow \; \swarrow \; \searrow$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots$	4^i	$\frac{n}{2^i}$	$c(\frac{n}{2^i})$	$2^i cn$

$\frac{n}{2^i} = 1$ when $i = \log_2 n \implies$ Depth: $d = \log_2 n$.

$4^{\log n}$

Solving recurrences.

$$T(n) = 4T(\frac{n}{2}) + cn; \qquad T(1) = c$$

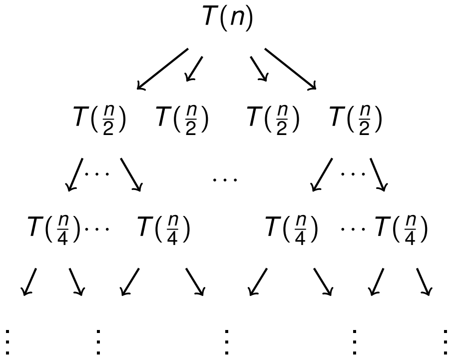
Recursion Tree	# probs	sz	time/prob	time/level
$T(n)$	1	n	cn	cn
$\swarrow \; \swarrow \quad \searrow \; \searrow$				
$T(\frac{n}{2}) \quad T(\frac{n}{2}) \quad T(\frac{n}{2}) \quad T(\frac{n}{2})$	4	$\frac{n}{2}$	$c(\frac{n}{2})$	$2cn$
$\swarrow \cdots \searrow \quad \cdots \quad \swarrow \cdots \searrow$				
$T(\frac{n}{4}) \cdots \; T(\frac{n}{4}) \quad T(\frac{n}{4}) \; \cdots T(\frac{n}{4})$	4^2	$\frac{n}{4}$	$c(\frac{n}{4})$	$4cn$
$\swarrow \; \searrow \; \swarrow \; \searrow \; \swarrow \; \searrow \; \swarrow \; \searrow$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots$	4^i	$\frac{n}{2^i}$	$c(\frac{n}{2^i})$	$2^i cn$

$\frac{n}{2^i} = 1$ when $i = \log_2 n \implies$ Depth: $d = \log_2 n$.
$4^{\log n} = 2^{2\log n}$

Solving recurrences.

$$T(n) = 4T(\frac{n}{2}) + cn; \qquad T(1) = c$$

Recursion Tree



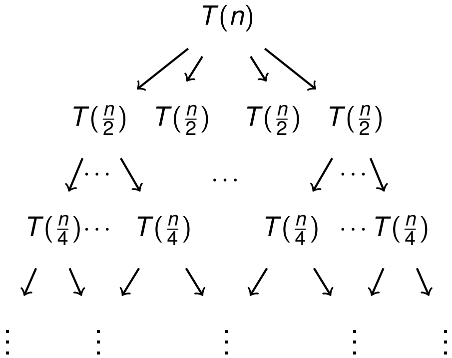
# probs	sz	time/prob	time/level
1	n	cn	cn
4	$\frac{n}{2}$	$c(\frac{n}{2})$	$2cn$
4^2	$\frac{n}{4}$	$c(\frac{n}{4})$	$4cn$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
4^i	$\frac{n}{2^i}$	$c(\frac{n}{2^i})$	$2^i cn$

$\frac{n}{2^i} = 1$ when $i = \log_2 n \implies$ Depth: $d = \log_2 n$.
$4^{\log n} = 2^{2\log n} = n^2$ base case problems.

Solving recurrences.

$$T(n) = 4T(\frac{n}{2}) + cn; \qquad T(1) = c$$

Recursion Tree



# probs	sz	time/prob	time/level
1	n	cn	cn
4	$\frac{n}{2}$	$c(\frac{n}{2})$	$2cn$
4^2	$\frac{n}{4}$	$c(\frac{n}{4})$	$4cn$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
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Recursion Tree	# probs	sz	time/prob	time/level
$T(n)$	1	n	cn	cn
$\swarrow \; \swarrow \quad \searrow \; \searrow$				
$T(\frac{n}{2}) \quad T(\frac{n}{2}) \quad T(\frac{n}{2}) \quad T(\frac{n}{2})$	4	$\frac{n}{2}$	$c(\frac{n}{2})$	$2cn$
$\swarrow \cdots \searrow \quad \cdots \quad \swarrow \cdots \searrow$				
$T(\frac{n}{4}) \cdots \quad T(\frac{n}{4}) \quad T(\frac{n}{4}) \quad \cdots T(\frac{n}{4})$	4^2	$\frac{n}{4}$	$c(\frac{n}{4})$	$4cn$
$\swarrow \; \searrow \; \swarrow \; \searrow \; \swarrow \; \searrow \; \swarrow \; \searrow$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots$	4^i	$\frac{n}{2^i}$	$c(\frac{n}{2^i})$	$2^i cn$

$\frac{n}{2^i} = 1$ when $i = \log_2 n \implies$ Depth: $d = \log_2 n$.

$4^{\log n} = 2^{2\log n} = n^2$ base case problems. size 1. Work/Prob: c

Solving recurrences.

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$T(n)$	1	n	cn	cn
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$T(\frac{n}{2}) \quad T(\frac{n}{2}) \quad T(\frac{n}{2}) \quad T(\frac{n}{2})$	4	$\frac{n}{2}$	$c(\frac{n}{2})$	$2cn$
$\swarrow \cdots \searrow \quad \cdots \quad \swarrow \cdots \searrow$				
$T(\frac{n}{4}) \cdots \quad T(\frac{n}{4}) \qquad T(\frac{n}{4}) \quad \cdots T(\frac{n}{4})$	4^2	$\frac{n}{4}$	$c(\frac{n}{4})$	$4cn$
$\swarrow \; \searrow \; \swarrow \; \searrow \; \swarrow \; \searrow \; \swarrow \; \searrow$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots$	4^i	$\frac{n}{2^i}$	$c(\frac{n}{2^i})$	$2^i cn$

$\frac{n}{2^i} = 1$ when $i = \log_2 n \implies$ Depth: $d = \log_2 n$.
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Work: cn^2.

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$\swarrow \; \swarrow \quad \searrow \; \searrow$				
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$\swarrow \cdots \searrow \quad \cdots \quad \swarrow \cdots \searrow$				
$T(\frac{n}{4}) \cdots \; T(\frac{n}{4}) \quad T(\frac{n}{4}) \; \cdots T(\frac{n}{4})$	4^2	$\frac{n}{4}$	$c(\frac{n}{4})$	$4cn$
$\swarrow \; \searrow \; \swarrow \; \searrow \; \swarrow \; \searrow \; \swarrow \; \searrow$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots$	4^i	$\frac{n}{2^i}$	$c(\frac{n}{2^i})$	$2^i cn$

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Work: cn^2.
Total Work:

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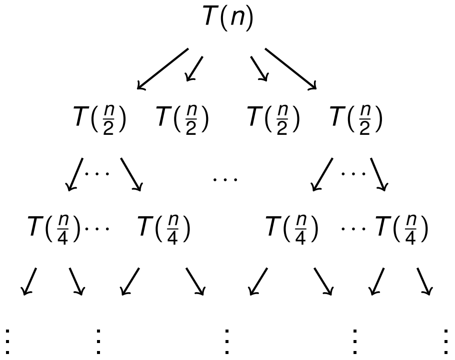
$T(\frac{n}{4})\cdots$ $T(\frac{n}{4})$ $T(\frac{n}{4})$ $\cdots T(\frac{n}{4})$	4^2	$\frac{n}{4}$	$c(\frac{n}{4})$	$4cn$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\vdots$	4^i	$\frac{n}{2^i}$	$c(\frac{n}{2^i})$	$2^i cn$

$\frac{n}{2^i} = 1$ when $i = \log_2 n \implies$ Depth: $d = \log_2 n$.
$4^{\log n} = 2^{2\log n} = n^2$ base case problems. size 1. Work/Prob: c
Work: cn^2.
Total Work: cn

Solving recurrences.

$$T(n) = 4T(\frac{n}{2}) + cn; \qquad T(1) = c$$

Recursion Tree



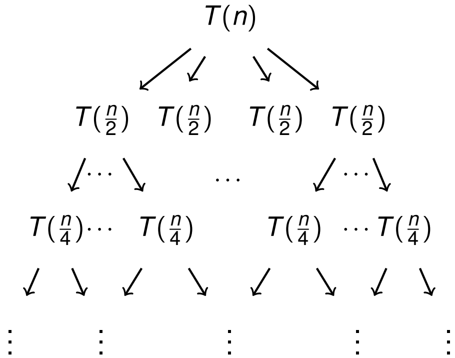
# probs	sz	time/prob	time/level
1	n	cn	cn
4	$\frac{n}{2}$	$c(\frac{n}{2})$	$2cn$
4^2	$\frac{n}{4}$	$c(\frac{n}{4})$	$4cn$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
4^i	$\frac{n}{2^i}$	$c(\frac{n}{2^i})$	$2^i cn$

$\frac{n}{2^i} = 1$ when $i = \log_2 n \implies$ Depth: $d = \log_2 n$.
$4^{\log n} = 2^{2\log n} = n^2$ base case problems. size 1. Work/Prob: c
Work: cn^2.
Total Work: $cn + 2cn$

Solving recurrences.

$$T(n) = 4T(\frac{n}{2}) + cn; \qquad T(1) = c$$

Recursion Tree



# probs	sz	time/prob	time/level
1	n	cn	cn
4	$\frac{n}{2}$	$c(\frac{n}{2})$	$2cn$
4^2	$\frac{n}{4}$	$c(\frac{n}{4})$	$4cn$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
4^i	$\frac{n}{2^i}$	$c(\frac{n}{2^i})$	$2^i cn$

$\frac{n}{2^i} = 1$ when $i = \log_2 n \implies$ Depth: $d = \log_2 n$.
$4^{\log n} = 2^{2\log n} = n^2$ base case problems. size 1. Work/Prob: c
Work: cn^2.
Total Work: $cn + 2cn + 4cn + \cdots$

Solving recurrences.

$$T(n) = 4T(\frac{n}{2}) + cn; \qquad T(1) = c$$

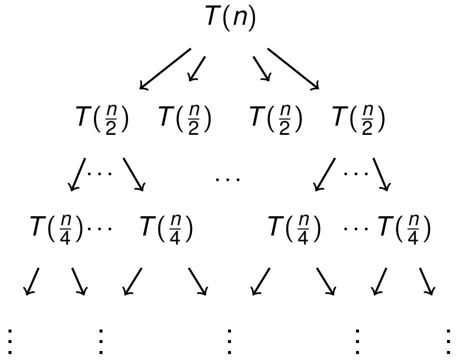
Recursion Tree	# probs	sz	time/prob	time/level
$T(n)$	1	n	cn	cn
$\swarrow \; \swarrow \quad \searrow \; \searrow$				
$T(\frac{n}{2}) \quad T(\frac{n}{2}) \quad T(\frac{n}{2}) \quad T(\frac{n}{2})$	4	$\frac{n}{2}$	$c(\frac{n}{2})$	$2cn$
$\swarrow \cdots \searrow \quad \cdots \quad \swarrow \cdots \searrow$				
$T(\frac{n}{4}) \cdots \quad T(\frac{n}{4}) \quad T(\frac{n}{4}) \quad \cdots T(\frac{n}{4})$	4^2	$\frac{n}{4}$	$c(\frac{n}{4})$	$4cn$
$\swarrow \; \searrow \; \swarrow \; \searrow \; \swarrow \; \searrow \; \swarrow \; \searrow$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots$	4^i	$\frac{n}{2^i}$	$c(\frac{n}{2^i})$	$2^i cn$

$\frac{n}{2^i} = 1$ when $i = \log_2 n \implies$ Depth: $d = \log_2 n$.
$4^{\log n} = 2^{2\log n} = n^2$ base case problems. size 1. Work/Prob: c
Work: cn^2.
Total Work: $cn + 2cn + 4cn + \cdots + cn^2$

Solving recurrences.

$$T(n) = 4T(\frac{n}{2}) + cn; \qquad T(1) = c$$

Recursion Tree



# probs	sz	time/prob	time/level
1	n	cn	cn
4	$\frac{n}{2}$	$c(\frac{n}{2})$	$2cn$
4^2	$\frac{n}{4}$	$c(\frac{n}{4})$	$4cn$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
4^i	$\frac{n}{2^i}$	$c(\frac{n}{2^i})$	$2^i cn$

$\frac{n}{2^i} = 1$ when $i = \log_2 n \implies$ Depth: $d = \log_2 n$.

$4^{\log n} = 2^{2\log n} = n^2$ base case problems. size 1. Work/Prob: c

Work: cn^2.

Total Work: $cn + 2cn + 4cn + \cdots + cn^2 = O(n^2)$.

Solving recurrences.

$$T(n) = 4T(\frac{n}{2}) + cn; \qquad T(1) = c$$

Recursion Tree	# probs	sz	time/prob	time/level
$T(n)$	1	n	cn	cn
$\swarrow \; \swarrow \quad \searrow \; \searrow$				
$T(\frac{n}{2}) \; T(\frac{n}{2}) \; T(\frac{n}{2}) \; T(\frac{n}{2})$	4	$\frac{n}{2}$	$c(\frac{n}{2})$	$2cn$
$\swarrow \cdots \searrow \quad \cdots \quad \swarrow \cdots \searrow$				
$T(\frac{n}{4}) \cdots \; T(\frac{n}{4}) \quad T(\frac{n}{4}) \; \cdots T(\frac{n}{4})$	4^2	$\frac{n}{4}$	$c(\frac{n}{4})$	$4cn$
$\swarrow \; \searrow \; \swarrow \; \searrow \; \swarrow \; \searrow \; \swarrow \; \searrow$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots$	4^i	$\frac{n}{2^i}$	$c(\frac{n}{2^i})$	$2^i cn$

$\frac{n}{2^i} = 1$ when $i = \log_2 n \implies$ Depth: $d = \log_2 n$.
$4^{\log n} = 2^{2\log n} = n^2$ base case problems. size 1. Work/Prob: c
Work: cn^2.
Total Work: $cn + 2cn + 4cn + \cdots + cn^2 = O(n^2)$. Geometric series.

Solving recurrences.

$$T(n) = 4T(\frac{n}{2}) + cn; \qquad T(1) = c$$

Solving recurrences.

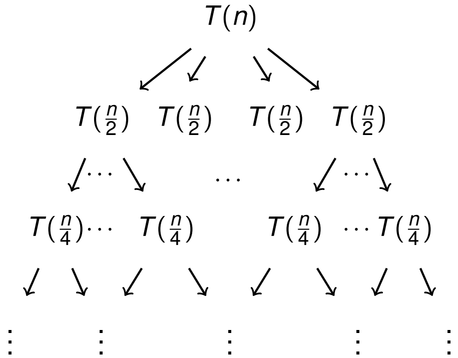
$$T(n) = 4T(\frac{n}{2}) + cn; \qquad T(1) = c$$

Recursion Tree	# probs	sz	time/prob	time/level
$T(n)$	1	n	cn	cn
$\swarrow \swarrow \searrow \searrow$				
$T(\frac{n}{2})$ $T(\frac{n}{2})$ $T(\frac{n}{2})$ $T(\frac{n}{2})$	4	$\frac{n}{2}$	$c(\frac{n}{2})$	$2cn$
$\swarrow \cdots \searrow$ $\cdots$ $\swarrow \cdots \searrow$				
$T(\frac{n}{4}) \cdots$ $T(\frac{n}{4})$ $T(\frac{n}{4})$ $\cdots T(\frac{n}{4})$	4^2	$\frac{n}{4}$	$c(\frac{n}{4})$	$4cn$
$\swarrow \searrow \swarrow \searrow \swarrow \searrow \swarrow \searrow$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\vdots$ $\vdots$ $\vdots$ $\vdots$ $\vdots$	4^i	$\frac{n}{2^i}$	$c(\frac{n}{2^i})$	$2^i cn$

Solving recurrences.

$$T(n) = 4T(\frac{n}{2}) + cn; \qquad T(1) = c$$

Recursion Tree



# probs	sz	time/prob	time/level
1	n	cn	cn
4	$\frac{n}{2}$	$c(\frac{n}{2})$	$2cn$
4^2	$\frac{n}{4}$	$c(\frac{n}{4})$	$4cn$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
4^i	$\frac{n}{2^i}$	$c(\frac{n}{2^i})$	$2^i cn$

$\frac{n}{2^i} = 1$ when $i = \log_2 n$

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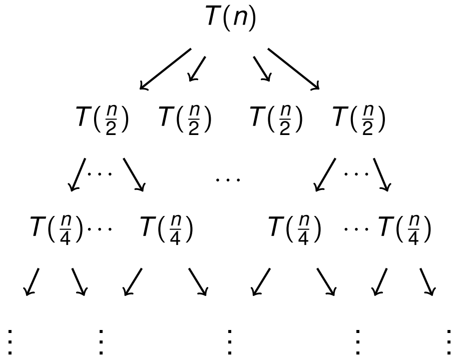
Recursion Tree	# probs	sz	time/prob	time/level
$T(n)$	1	n	cn	cn
$\swarrow \; \swarrow \quad \searrow \; \searrow$				
$T(\frac{n}{2}) \quad T(\frac{n}{2}) \quad T(\frac{n}{2}) \quad T(\frac{n}{2})$	4	$\frac{n}{2}$	$c(\frac{n}{2})$	$2cn$
$\swarrow \cdots \searrow \quad \cdots \quad \swarrow \cdots \searrow$				
$T(\frac{n}{4}) \cdots \; T(\frac{n}{4}) \quad T(\frac{n}{4}) \; \cdots T(\frac{n}{4})$	4^2	$\frac{n}{4}$	$c(\frac{n}{4})$	$4cn$
$\swarrow \; \searrow \; \swarrow \; \searrow \; \swarrow \; \searrow \; \swarrow \; \searrow$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots$	4^i	$\frac{n}{2^i}$	$c(\frac{n}{2^i})$	$2^i cn$

$\frac{n}{2^i} = 1$ when $i = \log_2 n \implies$ Depth: $d = \log_2 n$.

Solving recurrences.

$$T(n) = 4T(\frac{n}{2}) + cn; \qquad T(1) = c$$

Recursion Tree



# probs	sz	time/prob	time/level
1	n	cn	cn
4	$\frac{n}{2}$	$c(\frac{n}{2})$	$2cn$
4^2	$\frac{n}{4}$	$c(\frac{n}{4})$	$4cn$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
4^i	$\frac{n}{2^i}$	$c(\frac{n}{2^i})$	$2^i cn$

$\frac{n}{2^i} = 1$ when $i = \log_2 n \implies$ Depth: $d = \log_2 n$.

$4^{\log n}$

Solving recurrences.

$$T(n) = 4T(\frac{n}{2}) + cn; \qquad T(1) = c$$

Recursion Tree	# probs	sz	time/prob	time/level
$T(n)$	1	n	cn	cn
$T(\frac{n}{2})$ $T(\frac{n}{2})$ $T(\frac{n}{2})$ $T(\frac{n}{2})$	4	$\frac{n}{2}$	$c(\frac{n}{2})$	$2cn$
$T(\frac{n}{4})\cdots$ $T(\frac{n}{4})$ $T(\frac{n}{4})$ $\cdots T(\frac{n}{4})$	4^2	$\frac{n}{4}$	$c(\frac{n}{4})$	$4cn$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\vdots$	4^i	$\frac{n}{2^i}$	$c(\frac{n}{2^i})$	$2^i cn$

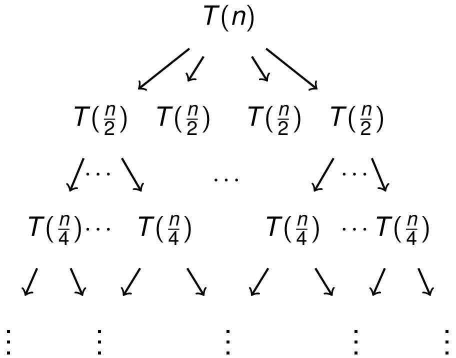
$\frac{n}{2^i} = 1$ when $i = \log_2 n \implies$ Depth: $d = \log_2 n$.

$4^{\log n} = 2^{2\log n}$

Solving recurrences.

$$T(n) = 4T(\frac{n}{2}) + cn; \qquad T(1) = c$$

Recursion Tree



# probs	sz	time/prob	time/level
1	n	cn	cn
4	$\frac{n}{2}$	$c(\frac{n}{2})$	$2cn$
4^2	$\frac{n}{4}$	$c(\frac{n}{4})$	$4cn$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
4^i	$\frac{n}{2^i}$	$c(\frac{n}{2^i})$	$2^i cn$

$\frac{n}{2^i} = 1$ when $i = \log_2 n \implies$ Depth: $d = \log_2 n$.
$4^{\log n} = 2^{2\log n} = n^2$ base case problems.

Solving recurrences.

$$T(n) = 4T(\frac{n}{2}) + cn; \qquad T(1) = c$$

Recursion Tree	# probs	sz	time/prob	time/level
$T(n)$	1	n	cn	cn
$\swarrow \swarrow \searrow \searrow$				
$T(\frac{n}{2})$ $T(\frac{n}{2})$ $T(\frac{n}{2})$ $T(\frac{n}{2})$	4	$\frac{n}{2}$	$c(\frac{n}{2})$	$2cn$
$\swarrow \cdots \searrow$ $\cdots$ $\swarrow \cdots \searrow$				
$T(\frac{n}{4}) \cdots$ $T(\frac{n}{4})$ $T(\frac{n}{4})$ $\cdots T(\frac{n}{4})$	4^2	$\frac{n}{4}$	$c(\frac{n}{4})$	$4cn$
$\swarrow \searrow \swarrow \searrow \swarrow \searrow \swarrow \searrow$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\vdots$ $\vdots$ $\vdots$ $\vdots$ $\vdots$	4^i	$\frac{n}{2^i}$	$c(\frac{n}{2^i})$	$2^i cn$

$\frac{n}{2^i} = 1$ when $i = \log_2 n \implies$ Depth: $d = \log_2 n$.
$4^{\log n} = 2^{2\log n} = n^2$ base case problems. size 1.

Solving recurrences.

$$T(n) = 4T(\frac{n}{2}) + cn; \qquad T(1) = c$$

Recursion Tree	# probs	sz	time/prob	time/level
$T(n)$	1	n	cn	cn
$\swarrow \swarrow \searrow \searrow$				
$T(\frac{n}{2})$ $T(\frac{n}{2})$ $T(\frac{n}{2})$ $T(\frac{n}{2})$	4	$\frac{n}{2}$	$c(\frac{n}{2})$	$2cn$
$\swarrow \cdots \searrow \quad \cdots \quad \swarrow \cdots \searrow$				
$T(\frac{n}{4}) \cdots T(\frac{n}{4})$ $T(\frac{n}{4}) \cdots T(\frac{n}{4})$	4^2	$\frac{n}{4}$	$c(\frac{n}{4})$	$4cn$
$\swarrow \searrow \swarrow \searrow \swarrow \searrow \swarrow \searrow$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots$	4^i	$\frac{n}{2^i}$	$c(\frac{n}{2^i})$	$2^i cn$

$\frac{n}{2^i} = 1$ when $i = \log_2 n \implies$ Depth: $d = \log_2 n$.
$4^{\log n} = 2^{2\log n} = n^2$ base case problems. size 1. Work/Prob: c

Solving recurrences.

$$T(n) = 4T(\frac{n}{2}) + cn; \qquad T(1) = c$$

Recursion Tree	# probs	sz	time/prob	time/level
$T(n)$	1	n	cn	cn
$\swarrow \; \swarrow \quad \searrow \; \searrow$				
$T(\frac{n}{2}) \quad T(\frac{n}{2}) \quad T(\frac{n}{2}) \quad T(\frac{n}{2})$	4	$\frac{n}{2}$	$c(\frac{n}{2})$	$2cn$
$\swarrow \cdots \searrow \quad \cdots \quad \swarrow \cdots \searrow$				
$T(\frac{n}{4}) \cdots \quad T(\frac{n}{4}) \quad T(\frac{n}{4}) \quad \cdots T(\frac{n}{4})$	4^2	$\frac{n}{4}$	$c(\frac{n}{4})$	$4cn$
$\swarrow \; \searrow \; \swarrow \; \searrow \; \swarrow \; \searrow \; \swarrow \; \searrow$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots$	4^i	$\frac{n}{2^i}$	$c(\frac{n}{2^i})$	$2^i cn$

$\frac{n}{2^i} = 1$ when $i = \log_2 n \implies$ Depth: $d = \log_2 n$.
$4^{\log n} = 2^{2\log n} = n^2$ base case problems. size 1. Work/Prob: c
Work: cn^2.

Solving recurrences.

$$T(n) = 4T(\frac{n}{2}) + cn; \qquad T(1) = c$$

Recursion Tree	# probs	sz	time/prob	time/level
$T(n)$	1	n	cn	cn
$\swarrow \; \swarrow \quad \searrow \; \searrow$				
$T(\frac{n}{2}) \quad T(\frac{n}{2}) \quad T(\frac{n}{2}) \quad T(\frac{n}{2})$	4	$\frac{n}{2}$	$c(\frac{n}{2})$	$2cn$
$\swarrow \cdots \searrow \quad \cdots \quad \swarrow \cdots \searrow$				
$T(\frac{n}{4}) \cdots \quad T(\frac{n}{4}) \quad T(\frac{n}{4}) \quad \cdots T(\frac{n}{4})$	4^2	$\frac{n}{4}$	$c(\frac{n}{4})$	$4cn$
$\swarrow \; \searrow \; \swarrow \; \searrow \; \swarrow \; \searrow \; \swarrow \; \searrow$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots$	4^i	$\frac{n}{2^i}$	$c(\frac{n}{2^i})$	$2^i cn$

$\frac{n}{2^i} = 1$ when $i = \log_2 n \implies$ Depth: $d = \log_2 n$.

$4^{\log n} = 2^{2\log n} = n^2$ base case problems. size 1. Work/Prob: c

Work: cn^2.

Total Work:

Solving recurrences.

$$T(n) = 4T(\frac{n}{2}) + cn; \qquad T(1) = c$$

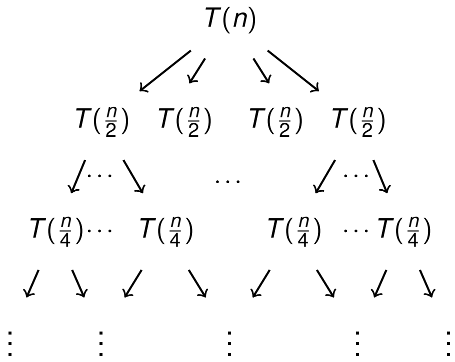
Recursion Tree	# probs	sz	time/prob	time/level
$T(n)$	1	n	cn	cn
$T(\frac{n}{2})$ $T(\frac{n}{2})$ $T(\frac{n}{2})$ $T(\frac{n}{2})$	4	$\frac{n}{2}$	$c(\frac{n}{2})$	$2cn$
$T(\frac{n}{4}) \cdots$ $T(\frac{n}{4})$ $T(\frac{n}{4})$ $\cdots T(\frac{n}{4})$	4^2	$\frac{n}{4}$	$c(\frac{n}{4})$	$4cn$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\vdots$	4^i	$\frac{n}{2^i}$	$c(\frac{n}{2^i})$	$2^i cn$

$\frac{n}{2^i} = 1$ when $i = \log_2 n \implies$ Depth: $d = \log_2 n$.
$4^{\log n} = 2^{2\log n} = n^2$ base case problems. size 1. Work/Prob: c
Work: cn^2.
Total Work: cn

Solving recurrences.

$$T(n) = 4T(\frac{n}{2}) + cn; \qquad T(1) = c$$

Recursion Tree



# probs	sz	time/prob	time/level
1	n	cn	cn
4	$\frac{n}{2}$	$c(\frac{n}{2})$	$2cn$
4^2	$\frac{n}{4}$	$c(\frac{n}{4})$	$4cn$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
4^i	$\frac{n}{2^i}$	$c(\frac{n}{2^i})$	$2^i cn$

$\frac{n}{2^i} = 1$ when $i = \log_2 n \implies$ Depth: $d = \log_2 n$.
$4^{\log n} = 2^{2\log n} = n^2$ base case problems. size 1. Work/Prob: c
Work: cn^2.
Total Work: $cn + 2cn$

Solving recurrences.

$$T(n) = 4T(\frac{n}{2}) + cn; \qquad T(1) = c$$

Recursion Tree	# probs	sz	time/prob	time/level
$T(n)$	1	n	cn	cn
$\swarrow \; \swarrow \quad \searrow \; \searrow$				
$T(\frac{n}{2}) \quad T(\frac{n}{2}) \quad T(\frac{n}{2}) \quad T(\frac{n}{2})$	4	$\frac{n}{2}$	$c(\frac{n}{2})$	$2cn$
$\swarrow \cdots \searrow \quad \cdots \quad \swarrow \cdots \searrow$				
$T(\frac{n}{4}) \cdots \quad T(\frac{n}{4}) \qquad T(\frac{n}{4}) \quad \cdots T(\frac{n}{4})$	4^2	$\frac{n}{4}$	$c(\frac{n}{4})$	$4cn$
$\swarrow \; \searrow \; \swarrow \; \searrow \; \swarrow \; \searrow \; \swarrow \; \searrow$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots$	4^i	$\frac{n}{2^i}$	$c(\frac{n}{2^i})$	$2^i cn$

$\frac{n}{2^i} = 1$ when $i = \log_2 n \implies$ Depth: $d = \log_2 n$.
$4^{\log n} = 2^{2\log n} = n^2$ base case problems. size 1. Work/Prob: c
Work: cn^2.
Total Work: $cn + 2cn + 4cn + \cdots$

Solving recurrences.

$$T(n) = 4T(\frac{n}{2}) + cn; \qquad T(1) = c$$

Recursion Tree	# probs	sz	time/prob	time/level
$T(n)$	1	n	cn	cn
$T(\frac{n}{2})$ $T(\frac{n}{2})$ $T(\frac{n}{2})$ $T(\frac{n}{2})$	4	$\frac{n}{2}$	$c(\frac{n}{2})$	$2cn$
$T(\frac{n}{4})\cdots$ $T(\frac{n}{4})$ $T(\frac{n}{4})$ $\cdots T(\frac{n}{4})$	4^2	$\frac{n}{4}$	$c(\frac{n}{4})$	$4cn$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\vdots$	4^i	$\frac{n}{2^i}$	$c(\frac{n}{2^i})$	$2^i cn$

$\frac{n}{2^i} = 1$ when $i = \log_2 n \implies$ Depth: $d = \log_2 n$.
$4^{\log n} = 2^{2\log n} = n^2$ base case problems. size 1. Work/Prob: c
Work: cn^2.
Total Work: $cn + 2cn + 4cn + \cdots + cn^2$

Solving recurrences.

$$T(n) = 4T(\frac{n}{2}) + cn; \qquad T(1) = c$$

Recursion Tree	# probs	sz	time/prob	time/level
$T(n)$	1	n	cn	cn
$\swarrow \ \swarrow \quad \searrow \ \searrow$				
$T(\frac{n}{2}) \quad T(\frac{n}{2}) \quad T(\frac{n}{2}) \quad T(\frac{n}{2})$	4	$\frac{n}{2}$	$c(\frac{n}{2})$	$2cn$
$\swarrow \cdots \searrow \quad \cdots \quad \swarrow \cdots \searrow$				
$T(\frac{n}{4}) \cdots \quad T(\frac{n}{4}) \quad T(\frac{n}{4}) \quad \cdots T(\frac{n}{4})$	4^2	$\frac{n}{4}$	$c(\frac{n}{4})$	$4cn$
$\swarrow \ \searrow \ \swarrow \ \searrow \ \swarrow \ \searrow \ \swarrow \ \searrow$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots$	4^i	$\frac{n}{2^i}$	$c(\frac{n}{2^i})$	$2^i cn$

$\frac{n}{2^i} = 1$ when $i = \log_2 n \implies$ Depth: $d = \log_2 n$.

$4^{\log n} = 2^{2\log n} = n^2$ base case problems. size 1. Work/Prob: c

Work: cn^2.

Total Work: $cn + 2cn + 4cn + \cdots + cn^2 = O(n^2)$.

Solving recurrences.

$$T(n) = 4T(\frac{n}{2}) + cn; \qquad T(1) = c$$

Recursion Tree	# probs	sz	time/prob	time/level
$T(n)$	1	n	cn	cn
$\swarrow \ \swarrow \quad \searrow \ \searrow$				
$T(\frac{n}{2}) \quad T(\frac{n}{2}) \quad T(\frac{n}{2}) \quad T(\frac{n}{2})$	4	$\frac{n}{2}$	$c(\frac{n}{2})$	$2cn$
$\swarrow \cdots \searrow \quad \cdots \quad \swarrow \cdots \searrow$				
$T(\frac{n}{4}) \cdots \quad T(\frac{n}{4}) \quad T(\frac{n}{4}) \quad \cdots T(\frac{n}{4})$	4^2	$\frac{n}{4}$	$c(\frac{n}{4})$	$4cn$
$\swarrow \ \searrow \ \swarrow \ \searrow \ \swarrow \ \searrow \ \swarrow \ \searrow$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots$	4^i	$\frac{n}{2^i}$	$c(\frac{n}{2^i})$	$2^i cn$

$\frac{n}{2^i} = 1$ when $i = \log_2 n \implies$ Depth: $d = \log_2 n$.
$4^{\log n} = 2^{2\log n} = n^2$ base case problems. size 1. Work/Prob: c
Work: cn^2.
Total Work: $cn + 2cn + 4cn + \cdots + cn^2 = O(n^2)$. Geometric series.

Fast multiplication.

$$T(n) = 3T(\frac{n}{2}) + cn; \qquad T(1) = c$$

Fast multiplication.

$$T(n) = 3T(\frac{n}{2}) + cn; \qquad T(1) = c$$

Recursion Tree	# probs	sz	time/prob	time/level
$T(n)$	1	n	cn	cn
$\swarrow \quad \downarrow \quad \searrow$				
$T(\frac{n}{2}) \quad T(\frac{n}{2}) \quad T(\frac{n}{2})$	3	$\frac{n}{2}$	$c(\frac{n}{2})$	$(\frac{3}{2})cn$
$\swarrow \cdots \searrow \quad \cdots \quad \swarrow \cdots \searrow$				
$T(\frac{n}{4}) \cdots T(\frac{n}{4}) \quad T(\frac{n}{4}) \cdots T(\frac{n}{4})$	3^2	$\frac{n}{4}$	$c(\frac{n}{4})$	$(\frac{3}{2})^2 cn$
$\swarrow \; \searrow \; \swarrow \; \searrow \; \swarrow \; \searrow \; \swarrow \; \searrow$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots$	3^i	$\frac{n}{2^i}$	$c(\frac{n}{2^i})$	$(\frac{3}{2})^i cn$

Fast multiplication.

$$T(n) = 3T(\frac{n}{2}) + cn; \qquad T(1) = c$$

Recursion Tree	# probs	sz	time/prob	time/level
$T(n)$	1	n	cn	cn
$\swarrow \ \downarrow \ \searrow$				
$T(\frac{n}{2}) \quad T(\frac{n}{2}) \quad T(\frac{n}{2})$	3	$\frac{n}{2}$	$c(\frac{n}{2})$	$(\frac{3}{2})cn$
$\swarrow \cdots \searrow \quad \cdots \quad \swarrow \cdots \searrow$				
$T(\frac{n}{4}) \cdots T(\frac{n}{4}) \quad T(\frac{n}{4}) \cdots T(\frac{n}{4})$	3^2	$\frac{n}{4}$	$c(\frac{n}{4})$	$(\frac{3}{2})^2 cn$
$\swarrow \ \searrow \ \swarrow \ \searrow \ \swarrow \ \searrow \ \swarrow \ \searrow$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots$	3^i	$\frac{n}{2^i}$	$c(\frac{n}{2^i})$	$(\frac{3}{2})^i cn$

$\frac{n}{2^i} = 1$ when $i = \log_2 n$

Fast multiplication.

$$T(n) = 3T(\frac{n}{2}) + cn; \qquad T(1) = c$$

Recursion Tree	# probs	sz	time/prob	time/level
$T(n)$	1	n	cn	cn
$\swarrow \quad \downarrow \quad \searrow$				
$T(\frac{n}{2}) \quad T(\frac{n}{2}) \quad T(\frac{n}{2})$	3	$\frac{n}{2}$	$c(\frac{n}{2})$	$(\frac{3}{2})cn$
$\swarrow \cdots \searrow \quad \cdots \quad \swarrow \cdots \searrow$				
$T(\frac{n}{4}) \cdots T(\frac{n}{4}) \quad T(\frac{n}{4}) \cdots T(\frac{n}{4})$	3^2	$\frac{n}{4}$	$c(\frac{n}{4})$	$(\frac{3}{2})^2 cn$
$\swarrow \ \searrow \ \swarrow \ \searrow \ \swarrow \ \searrow \ \swarrow \ \searrow$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots$	3^i	$\frac{n}{2^i}$	$c(\frac{n}{2^i})$	$(\frac{3}{2})^i cn$

$\frac{n}{2^i} = 1$ when $i = \log_2 n \implies$ Depth: $d = \log_2 n$.

Fast multiplication.

$$T(n) = 3T(\frac{n}{2}) + cn; \qquad T(1) = c$$

Recursion Tree	# probs	sz	time/prob	time/level
$T(n)$	1	n	cn	cn
$\swarrow \; \downarrow \; \searrow$				
$T(\frac{n}{2}) \quad T(\frac{n}{2}) \quad T(\frac{n}{2})$	3	$\frac{n}{2}$	$c(\frac{n}{2})$	$(\frac{3}{2})cn$
$\swarrow \cdots \searrow \quad \cdots \quad \swarrow \cdots \searrow$				
$T(\frac{n}{4}) \cdots T(\frac{n}{4}) \quad T(\frac{n}{4}) \cdots T(\frac{n}{4})$	3^2	$\frac{n}{4}$	$c(\frac{n}{4})$	$(\frac{3}{2})^2 cn$
$\swarrow \; \searrow \; \swarrow \; \searrow \; \swarrow \; \searrow \; \swarrow \; \searrow$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots$	3^i	$\frac{n}{2^i}$	$c(\frac{n}{2^i})$	$(\frac{3}{2})^i cn$

$\frac{n}{2^i} = 1$ when $i = \log_2 n \implies$ Depth: $d = \log_2 n$.

$3^{\log_2 n}$

Fast multiplication.

$$T(n) = 3T(\frac{n}{2}) + cn; \qquad T(1) = c$$

Recursion Tree	# probs	sz	time/prob	time/level
$T(n)$	1	n	cn	cn
$\swarrow \ \downarrow \ \searrow$				
$T(\frac{n}{2}) \quad T(\frac{n}{2}) \quad T(\frac{n}{2})$	3	$\frac{n}{2}$	$c(\frac{n}{2})$	$(\frac{3}{2})cn$
$\swarrow \cdots \searrow \quad \cdots \quad \swarrow \cdots \searrow$				
$T(\frac{n}{4}) \cdots T(\frac{n}{4}) \quad T(\frac{n}{4}) \cdots T(\frac{n}{4})$	3^2	$\frac{n}{4}$	$c(\frac{n}{4})$	$(\frac{3}{2})^2 cn$
$\swarrow \ \searrow \ \swarrow \ \searrow \ \swarrow \ \searrow \ \swarrow \ \searrow$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots$	3^i	$\frac{n}{2^i}$	$c(\frac{n}{2^i})$	$(\frac{3}{2})^i cn$

$\frac{n}{2^i} = 1$ when $i = \log_2 n \implies$ Depth: $d = \log_2 n$.
$3^{\log_2 n} = n^{\log_2 3}$ base case problems.

Fast multiplication.

$$T(n) = 3T(\frac{n}{2}) + cn; \qquad T(1) = c$$

Recursion Tree	# probs	sz	time/prob	time/level
$T(n)$	1	n	cn	cn
$\swarrow \quad \downarrow \quad \searrow$				
$T(\frac{n}{2}) \quad T(\frac{n}{2}) \quad T(\frac{n}{2})$	3	$\frac{n}{2}$	$c(\frac{n}{2})$	$(\frac{3}{2})cn$
$\swarrow \cdots \searrow \quad \cdots \quad \swarrow \cdots \searrow$				
$T(\frac{n}{4}) \cdots T(\frac{n}{4}) \quad T(\frac{n}{4}) \cdots T(\frac{n}{4})$	3^2	$\frac{n}{4}$	$c(\frac{n}{4})$	$(\frac{3}{2})^2 cn$
$\swarrow \; \searrow \; \swarrow \; \searrow \; \swarrow \; \searrow \; \swarrow \; \searrow$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots$	3^i	$\frac{n}{2^i}$	$c(\frac{n}{2^i})$	$(\frac{3}{2})^i cn$

$\frac{n}{2^i} = 1$ when $i = \log_2 n \implies$ Depth: $d = \log_2 n$.
$3^{\log_2 n} = n^{\log_2 3}$ base case problems. size 1.

Fast multiplication.

$$T(n) = 3T(\frac{n}{2}) + cn; \qquad T(1) = c$$

Recursion Tree	# probs	sz	time/prob	time/level
$T(n)$	1	n	cn	cn
$\swarrow \quad \downarrow \quad \searrow$				
$T(\frac{n}{2}) \quad T(\frac{n}{2}) \quad T(\frac{n}{2})$	3	$\frac{n}{2}$	$c(\frac{n}{2})$	$(\frac{3}{2})cn$
$\swarrow \cdots \searrow \quad \cdots \quad \swarrow \cdots \searrow$				
$T(\frac{n}{4}) \cdots T(\frac{n}{4}) \quad T(\frac{n}{4}) \cdots T(\frac{n}{4})$	3^2	$\frac{n}{4}$	$c(\frac{n}{4})$	$(\frac{3}{2})^2 cn$
$\swarrow \ \searrow \ \swarrow \ \searrow \ \swarrow \ \searrow \ \swarrow \ \searrow$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots$	3^i	$\frac{n}{2^i}$	$c(\frac{n}{2^i})$	$(\frac{3}{2})^i cn$

$\frac{n}{2^i} = 1$ when $i = \log_2 n \implies$ Depth: $d = \log_2 n$.

$3^{\log_2 n} = n^{\log_2 3}$ base case problems. size 1. Work/Prob: c.

Fast multiplication.

$$T(n) = 3T(\frac{n}{2}) + cn; \qquad T(1) = c$$

Recursion Tree	# probs	sz	time/prob	time/level
$T(n)$	1	n	cn	cn
$\swarrow \quad \downarrow \quad \searrow$				
$T(\frac{n}{2}) \quad T(\frac{n}{2}) \quad T(\frac{n}{2})$	3	$\frac{n}{2}$	$c(\frac{n}{2})$	$(\frac{3}{2})cn$
$\swarrow \cdots \searrow \quad \cdots \quad \swarrow \cdots \searrow$				
$T(\frac{n}{4}) \cdots T(\frac{n}{4}) \quad T(\frac{n}{4}) \cdots T(\frac{n}{4})$	3^2	$\frac{n}{4}$	$c(\frac{n}{4})$	$(\frac{3}{2})^2 cn$
$\swarrow \ \searrow \ \swarrow \ \searrow \ \swarrow \ \searrow \ \swarrow \ \searrow$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots$	3^i	$\frac{n}{2^i}$	$c(\frac{n}{2^i})$	$(\frac{3}{2})^i cn$

$\frac{n}{2^i} = 1$ when $i = \log_2 n \implies$ Depth: $d = \log_2 n$.
$3^{\log_2 n} = n^{\log_2 3}$ base case problems. size 1. Work/Prob: c. Work: $cn^{\log_2 3}$.

Fast multiplication.

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Recursion Tree	# probs	sz	time/prob	time/level
$T(n)$	1	n	cn	cn
$\swarrow \quad \downarrow \quad \searrow$				
$T(\frac{n}{2}) \quad T(\frac{n}{2}) \quad T(\frac{n}{2})$	3	$\frac{n}{2}$	$c(\frac{n}{2})$	$(\frac{3}{2})cn$
$\swarrow \cdots \searrow \quad \cdots \quad \swarrow \cdots \searrow$				
$T(\frac{n}{4}) \cdots T(\frac{n}{4}) \quad T(\frac{n}{4}) \cdots T(\frac{n}{4})$	3^2	$\frac{n}{4}$	$c(\frac{n}{4})$	$(\frac{3}{2})^2 cn$
$\swarrow \; \searrow \; \swarrow \; \searrow \; \swarrow \; \searrow \; \swarrow \; \searrow$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots$	3^i	$\frac{n}{2^i}$	$c(\frac{n}{2^i})$	$(\frac{3}{2})^i cn$

$\frac{n}{2^i} = 1$ when $i = \log_2 n \implies$ Depth: $d = \log_2 n$.
$3^{\log_2 n} = n^{\log_2 3}$ base case problems. size 1. Work/Prob: c. Work: $cn^{\log_2 3}$.
Total Work:

Fast multiplication.

$$T(n) = 3T(\frac{n}{2}) + cn; \qquad T(1) = c$$

Recursion Tree	# probs	sz	time/prob	time/level
$T(n)$	1	n	cn	cn
$\swarrow \ \downarrow \ \searrow$				
$T(\frac{n}{2}) \quad T(\frac{n}{2}) \quad T(\frac{n}{2})$	3	$\frac{n}{2}$	$c(\frac{n}{2})$	$(\frac{3}{2})cn$
$\swarrow \cdots \searrow \quad \cdots \quad \swarrow \cdots \searrow$				
$T(\frac{n}{4}) \cdots T(\frac{n}{4}) \quad T(\frac{n}{4}) \cdots T(\frac{n}{4})$	3^2	$\frac{n}{4}$	$c(\frac{n}{4})$	$(\frac{3}{2})^2 cn$
$\swarrow \ \searrow \ \swarrow \ \searrow \ \swarrow \ \searrow \ \swarrow \ \searrow$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots$	3^i	$\frac{n}{2^i}$	$c(\frac{n}{2^i})$	$(\frac{3}{2})^i cn$

$\frac{n}{2^i} = 1$ when $i = \log_2 n \implies$ Depth: $d = \log_2 n$.

$3^{\log_2 n} = n^{\log_2 3}$ base case problems. size 1. Work/Prob: c. Work: $cn^{\log_2 3}$.

Total Work: cn

Fast multiplication.

$$T(n) = 3T(\frac{n}{2}) + cn; \qquad T(1) = c$$

Recursion Tree	# probs	sz	time/prob	time/level
$T(n)$	1	n	cn	cn
$\swarrow \ \downarrow \ \searrow$				
$T(\frac{n}{2}) \quad T(\frac{n}{2}) \quad T(\frac{n}{2})$	3	$\frac{n}{2}$	$c(\frac{n}{2})$	$(\frac{3}{2})cn$
$\swarrow \cdots \searrow \quad \cdots \quad \swarrow \cdots \searrow$				
$T(\frac{n}{4}) \cdots T(\frac{n}{4}) \quad T(\frac{n}{4}) \cdots T(\frac{n}{4})$	3^2	$\frac{n}{4}$	$c(\frac{n}{4})$	$(\frac{3}{2})^2 cn$
$\swarrow \ \searrow \ \swarrow \ \searrow \ \swarrow \ \searrow \ \swarrow \ \searrow$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots$	3^i	$\frac{n}{2^i}$	$c(\frac{n}{2^i})$	$(\frac{3}{2})^i cn$

$\frac{n}{2^i} = 1$ when $i = \log_2 n \implies$ Depth: $d = \log_2 n$.
$3^{\log_2 n} = n^{\log_2 3}$ base case problems. size 1. Work/Prob: c. Work: $cn^{\log_2 3}$.
Total Work: $cn + (\frac{3}{2})cn$

Fast multiplication.

$$T(n) = 3T(\frac{n}{2}) + cn; \qquad T(1) = c$$

Recursion Tree	# probs	sz	time/prob	time/level
$T(n)$	1	n	cn	cn
$\swarrow \ \downarrow \ \searrow$				
$T(\frac{n}{2}) \quad T(\frac{n}{2}) \quad T(\frac{n}{2})$	3	$\frac{n}{2}$	$c(\frac{n}{2})$	$(\frac{3}{2})cn$
$\swarrow\cdots\searrow \quad \cdots \quad \swarrow\cdots\searrow$				
$T(\frac{n}{4})\cdots T(\frac{n}{4}) \quad T(\frac{n}{4})\cdots T(\frac{n}{4})$	3^2	$\frac{n}{4}$	$c(\frac{n}{4})$	$(\frac{3}{2})^2 cn$
$\swarrow \ \searrow \ \swarrow \ \searrow \ \swarrow \ \searrow \ \swarrow \ \searrow$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots$	3^i	$\frac{n}{2^i}$	$c(\frac{n}{2^i})$	$(\frac{3}{2})^i cn$

$\frac{n}{2^i} = 1$ when $i = \log_2 n \implies$ Depth: $d = \log_2 n$.

$3^{\log_2 n} = n^{\log_2 3}$ base case problems. size 1. Work/Prob: c. Work: $cn^{\log_2 3}$.

Total Work: $cn + (\frac{3}{2})cn + \cdots$

Fast multiplication.

$$T(n) = 3T(\frac{n}{2}) + cn; \qquad T(1) = c$$

Recursion Tree	# probs	sz	time/prob	time/level
$T(n)$	1	n	cn	cn
$\swarrow \quad \downarrow \quad \searrow$				
$T(\frac{n}{2}) \quad T(\frac{n}{2}) \quad T(\frac{n}{2})$	3	$\frac{n}{2}$	$c(\frac{n}{2})$	$(\frac{3}{2})cn$
$\swarrow \cdots \searrow \quad \cdots \quad \swarrow \cdots \searrow$				
$T(\frac{n}{4}) \cdots T(\frac{n}{4}) \quad T(\frac{n}{4}) \cdots T(\frac{n}{4})$	3^2	$\frac{n}{4}$	$c(\frac{n}{4})$	$(\frac{3}{2})^2 cn$
$\swarrow \; \searrow \; \swarrow \; \searrow \; \swarrow \; \searrow \; \swarrow \; \searrow$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots$	3^i	$\frac{n}{2^i}$	$c(\frac{n}{2^i})$	$(\frac{3}{2})^i cn$

$\frac{n}{2^i} = 1$ when $i = \log_2 n \implies$ Depth: $d = \log_2 n$.
$3^{\log_2 n} = n^{\log_2 3}$ base case problems. size 1. Work/Prob: c. Work: $cn^{\log_2 3}$.
Total Work: $cn + (\frac{3}{2})cn + \cdots + cn^{\log_2 3}$

Fast multiplication.

$$T(n) = 3T(\frac{n}{2}) + cn; \qquad T(1) = c$$

Recursion Tree	# probs	sz	time/prob	time/level
$T(n)$	1	n	cn	cn
$\swarrow \ \downarrow \ \searrow$				
$T(\frac{n}{2}) \quad T(\frac{n}{2}) \quad T(\frac{n}{2})$	3	$\frac{n}{2}$	$c(\frac{n}{2})$	$(\frac{3}{2})cn$
$\swarrow \cdots \searrow \quad \cdots \quad \swarrow \cdots \searrow$				
$T(\frac{n}{4}) \cdots T(\frac{n}{4}) \quad T(\frac{n}{4}) \cdots T(\frac{n}{4})$	3^2	$\frac{n}{4}$	$c(\frac{n}{4})$	$(\frac{3}{2})^2 cn$
$\swarrow \ \searrow \ \swarrow \ \searrow \ \swarrow \ \searrow \ \swarrow \ \searrow$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots$	3^i	$\frac{n}{2^i}$	$c(\frac{n}{2^i})$	$(\frac{3}{2})^i cn$

$\frac{n}{2^i} = 1$ when $i = \log_2 n \implies$ Depth: $d = \log_2 n$.
$3^{\log_2 n} = n^{\log_2 3}$ base case problems. size 1. Work/Prob: c. Work: $cn^{\log_2 3}$.
Total Work: $cn + (\frac{3}{2})cn + \cdots + cn^{\log_2 3} = O(n^{\log_2 3})$

Fast multiplication.

$$T(n) = 3T(\frac{n}{2}) + cn; \qquad T(1) = c$$

Recursion Tree	# probs	sz	time/prob	time/level
$T(n)$	1	n	cn	cn
$\swarrow \ \downarrow \ \searrow$				
$T(\frac{n}{2}) \quad T(\frac{n}{2}) \quad T(\frac{n}{2})$	3	$\frac{n}{2}$	$c(\frac{n}{2})$	$(\frac{3}{2})cn$
$\swarrow \cdots \searrow \quad \cdots \quad \swarrow \cdots \searrow$				
$T(\frac{n}{4}) \cdots T(\frac{n}{4}) \quad T(\frac{n}{4}) \cdots T(\frac{n}{4})$	3^2	$\frac{n}{4}$	$c(\frac{n}{4})$	$(\frac{3}{2})^2 cn$
$\swarrow \ \searrow \ \swarrow \ \searrow \ \swarrow \ \searrow \ \swarrow \ \searrow$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots$	3^i	$\frac{n}{2^i}$	$c(\frac{n}{2^i})$	$(\frac{3}{2})^i cn$

$\frac{n}{2^i} = 1$ when $i = \log_2 n \implies$ Depth: $d = \log_2 n$.
$3^{\log_2 n} = n^{\log_2 3}$ base case problems. size 1. Work/Prob: c. Work: $cn^{\log_2 3}$.
Total Work: $cn + (\frac{3}{2})cn + \cdots + cn^{\log_2 3} = O(n^{\log_2 3})$ Geometric series.

Divide and Conquer: In general.

$$T(n) = aT(\frac{n}{b}) + O(n^d); \qquad T(1) = c$$

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$$T(n) = aT(\frac{n}{b}) + O(n^d); \qquad T(1) = c$$

Recursion Tree	# probs	sz	time/prob	time/lvl
$T(n)$	1	n	cn^d	cn^d
$T(\frac{n}{b}) \quad T(\frac{n}{b}) \quad T(\frac{n}{b})$	a	$\frac{n}{b}$	$c(\frac{n}{b})^d$	$(\frac{a}{b^d})cn^d$
$T(\frac{n}{b^2}) \cdots T(\frac{n}{b^2}) \quad T(\frac{n}{b^2}) \cdots T(\frac{n}{b^2})$	a^2	$\frac{n}{b^2}$	$c(\frac{n}{b^2})^d$	$(\frac{a}{b^d})^2 cn^d$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\vdots$	a^i	$\frac{n}{b^i}$	$c(\frac{n}{b^i})^d$	$(\frac{a}{b^d})^i cn^d$

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Recursion Tree	# probs	sz	time/prob	time/lvl
$T(n)$	1	n	cn^d	cn^d
$\swarrow \quad \downarrow \quad \searrow$				
$T(\frac{n}{b}) \quad T(\frac{n}{b}) \quad T(\frac{n}{b})$	a	$\frac{n}{b}$	$c(\frac{n}{b})^d$	$(\frac{a}{b^d})cn^d$
$\swarrow \cdots \searrow \quad \cdots \quad \swarrow \cdots \searrow$				
$T(\frac{n}{b^2}) \cdots T(\frac{n}{b^2}) \quad T(\frac{n}{b^2}) \cdots T(\frac{n}{b^2})$	a^2	$\frac{n}{b^2}$	$c(\frac{n}{b^2})^d$	$(\frac{a}{b^d})^2 cn^d$
$\swarrow \; \searrow \; \swarrow \; \searrow \; \swarrow \; \searrow \; \swarrow \; \searrow$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots$	a^i	$\frac{n}{b^i}$	$c(\frac{n}{b^i})^d$	$(\frac{a}{b^d})^i cn^d$

$\frac{n}{b^i} = 1$ when $i = \log_b n$

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Recursion Tree	# probs	sz	time/prob	time/lvl
$T(n)$	1	n	cn^d	cn^d
$\swarrow \quad \downarrow \quad \searrow$				
$T(\frac{n}{b}) \quad T(\frac{n}{b}) \quad T(\frac{n}{b})$	a	$\frac{n}{b}$	$c(\frac{n}{b})^d$	$(\frac{a}{b^d})cn^d$
$\swarrow \cdots \searrow \quad \cdots \quad \swarrow \cdots \searrow$				
$T(\frac{n}{b^2}) \cdots T(\frac{n}{b^2}) \quad T(\frac{n}{b^2}) \cdots T(\frac{n}{b^2})$	a^2	$\frac{n}{b^2}$	$c(\frac{n}{b^2})^d$	$(\frac{a}{b^d})^2 cn^d$
$\swarrow \; \searrow \; \swarrow \; \searrow \; \swarrow \; \searrow \; \swarrow \; \searrow$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots$	a^i	$\frac{n}{b^i}$	$c(\frac{n}{b^i})^d$	$(\frac{a}{b^d})^i cn^d$

$\frac{n}{b^i} = 1$ when $i = \log_b n \implies$ Depth: $k = \log_b n$.

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$T(n)$	1	n	cn^d	cn^d
$\swarrow \quad \downarrow \quad \searrow$				
$T(\frac{n}{b}) \quad T(\frac{n}{b}) \quad T(\frac{n}{b})$	a	$\frac{n}{b}$	$c(\frac{n}{b})^d$	$(\frac{a}{b^d})cn^d$
$\swarrow \cdots \searrow \quad \cdots \quad \swarrow \cdots \searrow$				
$T(\frac{n}{b^2}) \cdots T(\frac{n}{b^2}) \quad T(\frac{n}{b^2}) \cdots T(\frac{n}{b^2})$	a^2	$\frac{n}{b^2}$	$c(\frac{n}{b^2})^d$	$(\frac{a}{b^d})^2 cn^d$
$\swarrow \ \searrow \ \swarrow \ \searrow \ \swarrow \ \searrow \ \swarrow \ \searrow$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots$	a^i	$\frac{n}{b^i}$	$c(\frac{n}{b^i})^d$	$(\frac{a}{b^d})^i cn^d$

$\frac{n}{b^i} = 1$ when $i = \log_b n \implies$ Depth: $k = \log_b n$.

Level i work: $(\frac{a}{b^d})^i n^d$.

Master's Theorem

Depth: $\log_b n$.

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Depth: $\log_b n$.
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Total:

$$n^d \sum_{i=0}^{\log_b n} (\frac{a}{b^d})^i$$

Master's Theorem

Depth: $\log_b n$.
Level i work:

$$(\frac{a}{b^d})^i n^d.$$

Total:

$$n^d \sum_{i=0}^{\log_b n} (\frac{a}{b^d})^i$$

Geometric series:

Master's Theorem

Depth: $\log_b n$.
Level i work:

$$(\frac{a}{b^d})^i n^d.$$

Total:

$$n^d \sum_{i=0}^{\log_b n} (\frac{a}{b^d})^i$$

Geometric series: If $\frac{a}{b^d} < 1$ ($d > \log_b a$), first term dominates

$$O(n^d),$$

Master's Theorem

Depth: $\log_b n$.
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if $\frac{a}{b^d} > 1$ ($d < \log_b a$), last term dominates.

$$O(n^{\log_b a}),$$

Master's Theorem

Depth: $\log_b n$.
Level i work:

$$(\frac{a}{b^d})^i n^d.$$

Total:

$$n^d \sum_{i=0}^{\log_b n} (\frac{a}{b^d})^i$$

Geometric series: If $\frac{a}{b^d} < 1$ ($d > \log_b a$), first term dominates

$$O(n^d),$$

if $\frac{a}{b^d} > 1$ ($d < \log_b a$), last term dominates.

$$O(n^{\log_b a}),$$

and if $\frac{a}{b^d} = 1$ ($d = \log_b a$), then all terms are the same

$$O(n^d \log_b n).$$

Master's Theorem: examples.

For a recurrence $T(n) = aT(n/b) + O(n^d)$
We have

$$
\begin{array}{ll}
d > \log_b a & T(n) = O(n^d) \\
d < \log_b a & T(n) = O(n^{\log_b a}) \\
d = \log_b a & T(n) = O(n^d \log_b n).
\end{array}
$$

Master's Theorem: examples.

For a recurrence $T(n) = aT(n/b) + O(n^d)$
We have

$d > \log_b a \quad T(n) = O(n^d)$
$d < \log_b a \quad T(n) = O(n^{\log_b a})$
$d = \log_b a \quad T(n) = O(n^d \log_b n).$

$T(n) = 4T(\frac{n}{2}) + O(n)$

Master's Theorem: examples.

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$T(n) = 4T(\frac{n}{2}) + O(n)$ $a = 4$, $b = 2$, and $d = 1$.

Master's Theorem: examples.

For a recurrence $T(n) = aT(n/b) + O(n^d)$
We have

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$d = \log_b a \qquad T(n) = O(n^d \log_b n)$.

$T(n) = 4T(\frac{n}{2}) + O(n)$ $a = 4$, $b = 2$, and $d = 1$.
$d = 1 < 2 = \log_2 4 = \log_b a$

Master's Theorem: examples.

For a recurrence $T(n) = aT(n/b) + O(n^d)$
We have

$d > \log_b a \quad T(n) = O(n^d)$
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$T(n) = 4T(\frac{n}{2}) + O(n)$ $a = 4$, $b = 2$, and $d = 1$.
$d = 1 < 2 = \log_2 4 = \log_b a \implies T(n) = O(n^{\log_b a}) = O(n^2)$.

Master's Theorem: examples.

For a recurrence $T(n) = aT(n/b) + O(n^d)$
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$d > \log_b a \qquad T(n) = O(n^d)$
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$T(n) = T(\frac{n}{2}) + O(n)$

Master's Theorem: examples.

For a recurrence $T(n) = aT(n/b) + O(n^d)$
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$d > \log_b a \quad T(n) = O(n^d)$
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Master's Theorem: examples.

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$T(n) = 4T(\frac{n}{2}) + O(n)$ $a = 4$, $b = 2$, and $d = 1$.
$d = 1 < 2 = \log_2 4 = \log_b a \implies T(n) = O(n^{\log_b a}) = O(n^2)$.

$T(n) = T(\frac{n}{2}) + O(n)$ $a = 1$, $b = 2$, and $d = 1$.
$1 > \log_2 1 = 0$

Master's Theorem: examples.

For a recurrence $T(n) = aT(n/b) + O(n^d)$
We have

$d > \log_b a \qquad T(n) = O(n^d)$
$d < \log_b a \qquad T(n) = O(n^{\log_b a})$
$d = \log_b a \qquad T(n) = O(n^d \log_b n)$.

$T(n) = 4T(\frac{n}{2}) + O(n)$ $a = 4$, $b = 2$, and $d = 1$.
$d = 1 < 2 = \log_2 4 = \log_b a \implies T(n) = O(n^{\log_b a}) = O(n^2)$.

$T(n) = T(\frac{n}{2}) + O(n)$ $a = 1$, $b = 2$, and $d = 1$.
$1 > \log_2 1 = 0 \implies T(n) = O(n)$

Master's Theorem: examples.

For a recurrence $T(n) = aT(n/b) + O(n^d)$
We have

$d > \log_b a \quad T(n) = O(n^d)$
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Strassen

Matrix multiplication.

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Strassen, 1968, visiting Berkeley.

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Strassen: Divide!

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Strassen: Divide! conquer!

Matrix Multiplication

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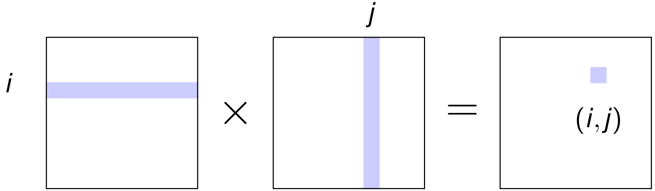

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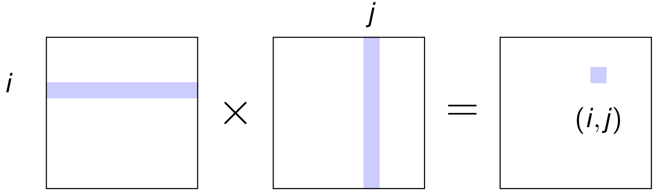

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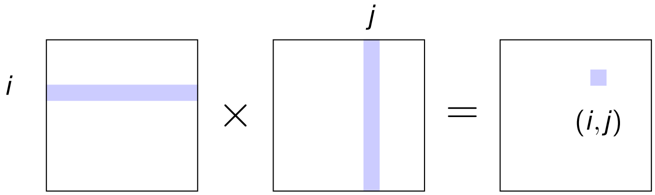

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Runtime?

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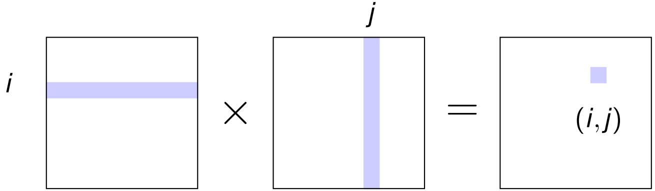

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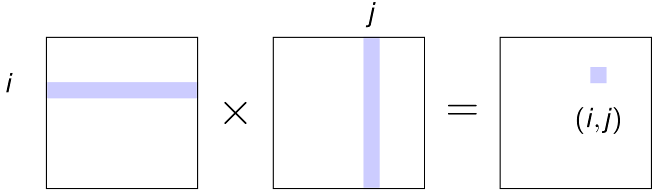

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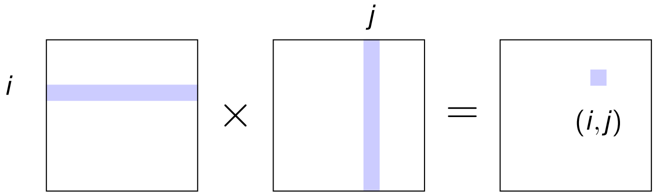

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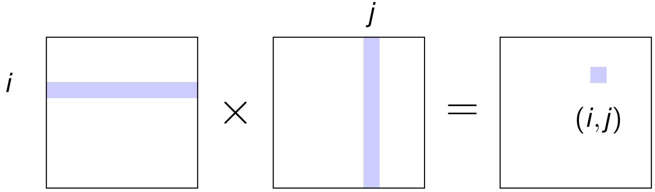

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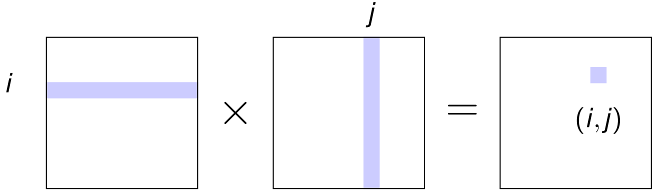

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Divide and Conquer

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} E & F \\ G & H \end{bmatrix} = \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}$$

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Masters: $O(n^{\log_2 8}) = O(n^3)$.

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(C) $O(n^{\log_2 7}) = O(n^{2.81\cdots})$

Strassen

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$= AE + BG$.

7 multiplies! Recurrence?

$T(n) = 7T(\frac{n}{2}) + O(n^2)$

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Commonly used in practice!

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