CS 170 Homework 1

Due **Monday 1/29/2024, at 10:00 pm (grace period until 11:59pm)**

1 Study Group

List the names and SIDs of the members in your study group. If you have no collaborators, you must explicitly write "none".

2 Course Policies

(a) What dates and times are the exams for CS170 this semester? Are there planned alternate exams?

(b) Homework is due Mondays at 10:00pm, with a late deadline at 11:59pm. At what time do we recommend you have your homework finished?

(c) We provide 2 homework drops for cases of emergency or technical issues that may arise due to homework submission. If you miss the Gradescope late deadline (even by a few minutes) and need to submit the homework, what should you do?

(d) What is the primary source of communication for CS170 to reach students? We will send out all important deadlines through this medium, and you are responsible for checking your emails and reading each announcement fully.

(e) Please read all of the following:

   (i) **Syllabus and Policies:** `https://cs170.org/syllabus/`

   (ii) **Homework Guidelines:** `https://cs170.org/resources/homework-guidelines/`

   (iii) **Regrade Etiquette:** `https://cs170.org/resources/regrade-etiquette/`

   (iv) **Forum Etiquette:** `https://cs170.org/resources/forum-etiquette/`

   Once you have read them, copy and sign the following sentence on your homework submission.

   "I have read and understood the course syllabus and policies."

3 Understanding Academic Dishonesty

Before you answer any of the following questions, make sure you have read over the syllabus and course policies (`https://cs170.org/syllabus/`) carefully. For each statement below, write *OK* if it is allowed by the course policies and *Not OK* otherwise.

(a) You ask a friend who took CS 170 previously for their homework solutions, some of which overlap with this semester's problem sets. You look at their solutions, then later write them down in your own words.

(b) You had 5 midterms on the same day and are behind on your homework. You decide to ask your classmate, who's already done the homework, for help. They tell you how to do the first three problems.

(c) You're a serial procrastinator and started working on the homework at 8:00 PM on Monday, and out of desperation searched up a homework problem online and find the exact solution. You then write it in your words and cite the source.

(d) You were looking up Dijkstra's on the internet, and inadvertently run into a website with a problem very similar to one on your homework. You read it, including the solution, and then you close the website, write up your solution, and cite the website URL in your homework writeup.

4 Math Potpourri

The following subparts will cover several math identities, tricks, and techniques that will be useful throughout the rest of this course.

(a) Simplify the following expressions into a single logarithm (i.e. in the form $\log_a b$):

 (i) $\frac{\ln x}{\ln y}$

 (ii) $\ln x + \ln y$

 (iii) $\ln x - \ln y$

 (iv) $170 \ln x$

(b) Give a simple proof for each of the following identities:

 (a) $x^{\log_{1/x} y} = \frac{1}{y}$

 (b) $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

 (c) $\sum_{k=0}^{n} ar^k = \begin{cases} a\left(\frac{1-r^{n+1}}{1-r}\right), & r \neq 1 \\ a(n+1), & r = 1 \end{cases}$

5 Recurrence Relations

For each part, find the asymptotic order of growth of T; that is, find a function g such that $T(n) = \Theta(g(n))$. Show your reasoning and **do not directly apply the Master Theorem; doing so will yield 0 credit**.

In all subparts, you may ignore any issues arising from whether a number is an integer.

(a) $T(n) = 2T(n/3) + 5n$

(b) $T(n) = 169T(n/170) + \Theta(n)$

(c) An algorithm $\mathcal{A}$ takes $\Theta(n^2)$ time to partition the input into 5 sub-problems of size $n/5$ each and then recursively runs itself on 3 of those subproblems. Describe the recurrence relation for the run-time $T(n)$ of $\mathcal{A}$ and find its asymptotic order of growth.

(d) $T(n) = 6T(n/6) + \Theta(n)$

(e) $T(n) = T(3n/5) + T(4n/5)$ (We have $T(1) = 1$)

Hint: first, compute a reasonable upper and lower bound for $T(n)$. Then, try to guess a $T(n)$ of the form an^b and then use induction to argue that it is correct.

6 In Between Functions

In this problem, we will find a function $f(n)$ that is asymptotically worse than polynomial time but still better than exponential time. In other words, f has to satisfy two properties,

- For all constants $k > 0$, $f(n) = \Omega(n^k)$ (1)
- For all constants $c > 0$, $f(n) = O(2^{cn})$ (2)

(a) Try setting $f(n)$ to a polynomial of degree d, where d is a very large constant. So $f(n) = a_0 + a_1 n + a_2 n^2 \cdots + a_d n^d$. For which values of k (if any) does f fail to satisfy (1)?

(b) Now try setting $f(n)$ to a^n, for some constant $a > 1$ that's as small as possible while still satisfying (1) (e.g. 1.000001). For which values of c (if any) does f fail to satisfy (2)?

Hint: Try rewriting a^n as 2^{bn} first, where b is a constant dependent on a.

So far we have found that the functions which look like $O(n^d)$ for constant d are too small and the functions that look like $O(a^n)$ are too large even if a is a tiny constant.

(c) Find a function $D(n)$ such that setting $f(n) = O(n^{D(n)})$ satisfies both (1) and (2). Give a proof that your answer satisfies both.

Hint: make sure $D(n)$ is asymptotically smaller than n.