Sorting and Hashing

See R&G Chapters:
9.1, 13.1-13.3, 13.4.2

Why Sort?

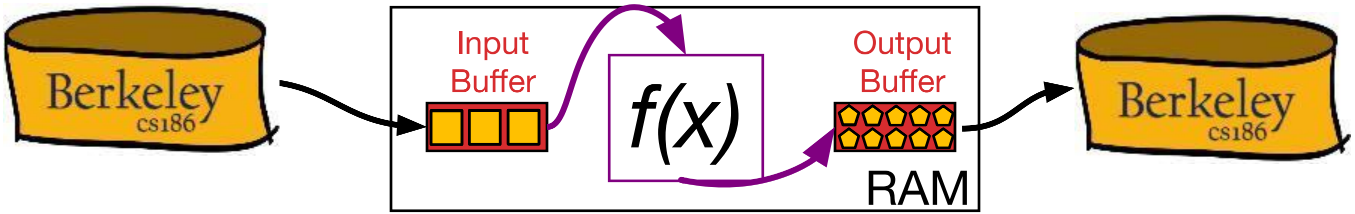
- “Rendezvous”
  - Eliminating duplicates (DISTINCT)
  - Grouping for summarization (GROUP BY)
  - Upcoming sort-merge join algorithm
- Ordering
  - Sometimes, output must be ordered (ORDER BY)
    - e.g., return results ranked in decreasing order of relevance
  - First step in bulk-loading tree indexes
- Problem: sort 100GB of data with 1GB of RAM.
  - why not virtual memory?

Out-of-Core Algorithms

- Two themes
  1. Single-pass streaming data through RAM
  2. Divide (into RAM-sized chunks) and Conquer

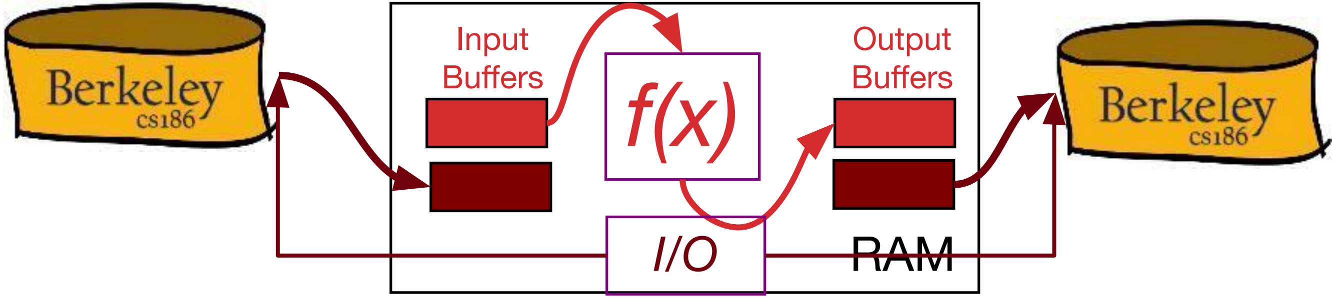
Single-pass Streaming

- Simple case: “Map”.
  - Goal: Compute f(x) for each record, write out the result
  - Challenge: minimize RAM, call read/write rarely
- Approach
  - Read a chunk from INPUT to an Input Buffer
  - Write f(x) for each item to an Output Buffer
  - When Input Buffer is consumed, read another chunk
  - When Output Buffer fills, write it to OUTPUT



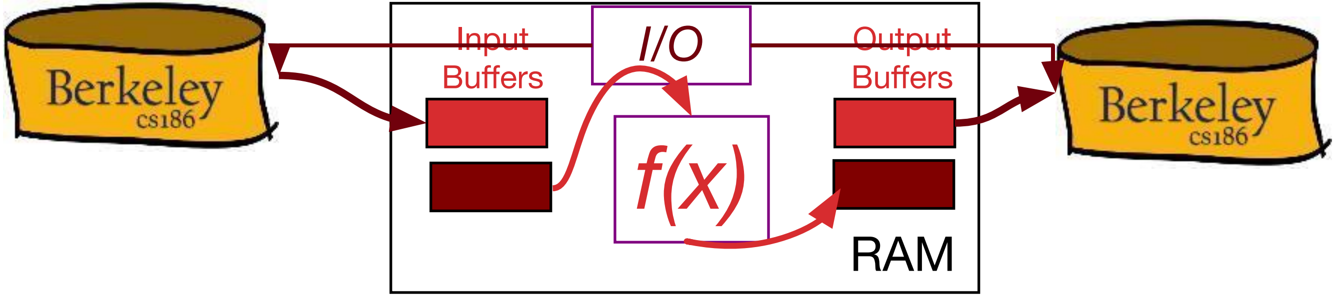
Better: Double Buffering pt 1

- **Main thread** runs f(x) on one pair I/O bufs
- 2nd **I/O thread** drains/fills unused I/O bufs in parallel
  - Why is parallelism available?
  - Theme: I/O handling usually deserves its own thread
- Main thread ready for a new buf? Swap!



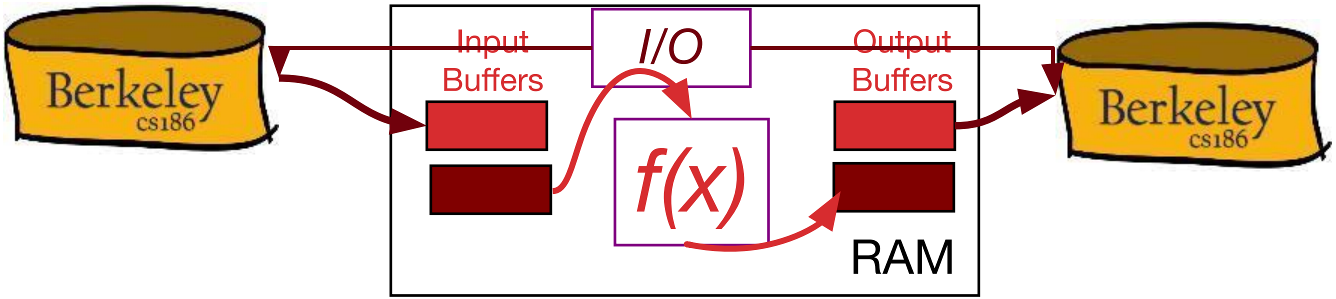
Better: Double Buffering pt 2

- **Main thread** runs f(x) on one pair I/O bufs
- 2nd **I/O thread** drains/fills unused I/O bufs in parallel
  - Why is parallelism available?
  - Theme: I/O handling usually deserves its own thread
- Main thread ready for a new buf? Swap!



Double Buffering applies to all streams

- Usable in any of the subsequent discussion
  - Assuming you have RAM buffers to spare!
  - But for simplicity we won't bring this up again.



Sorting & Hashing: Formal Specs

Sorting

- Produce an output file F_S
  - with contents R ***stored in order by a given sorting criterion***

Hashing

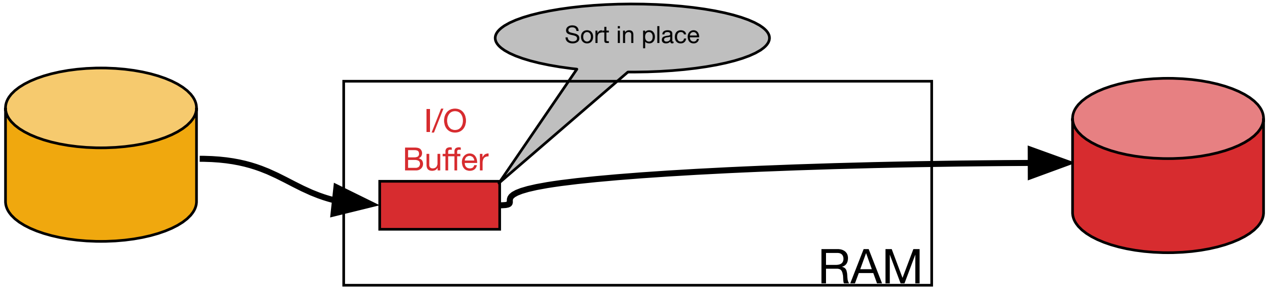
- Produce an output file F_H
  - with contents R, ***arranged on disk so that no 2 records that have the same hash value are separated by a record with a different hash value.***
  - I.e. matching records are always "stored consecutively" in F_H.

Given:

- A file F:
  - containing a multiset of records R
  - consuming **N** blocks of storage
- Two "scratch" disks
  - each with >> N blocks of free storage
- A fixed amount of space in RAM
  - memory capacity equivalent to **B** blocks of disk

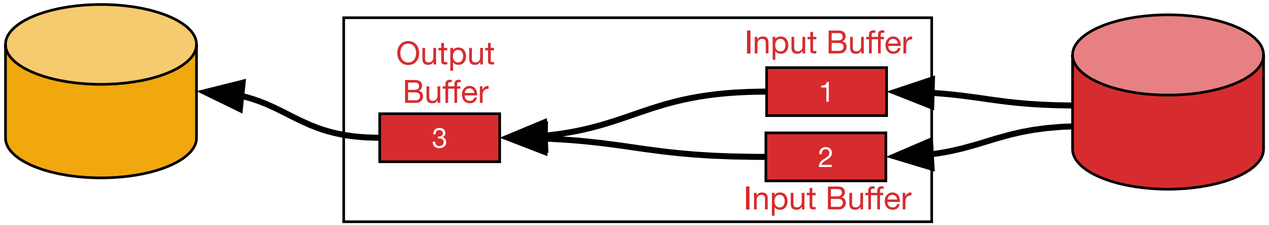
Sorting: 2-Way (a strawman)

- Pass 1 (conquer a batch):
  - read a page, sort it, write it.
  - only one buffer page is used
  - a repeated "batch job"

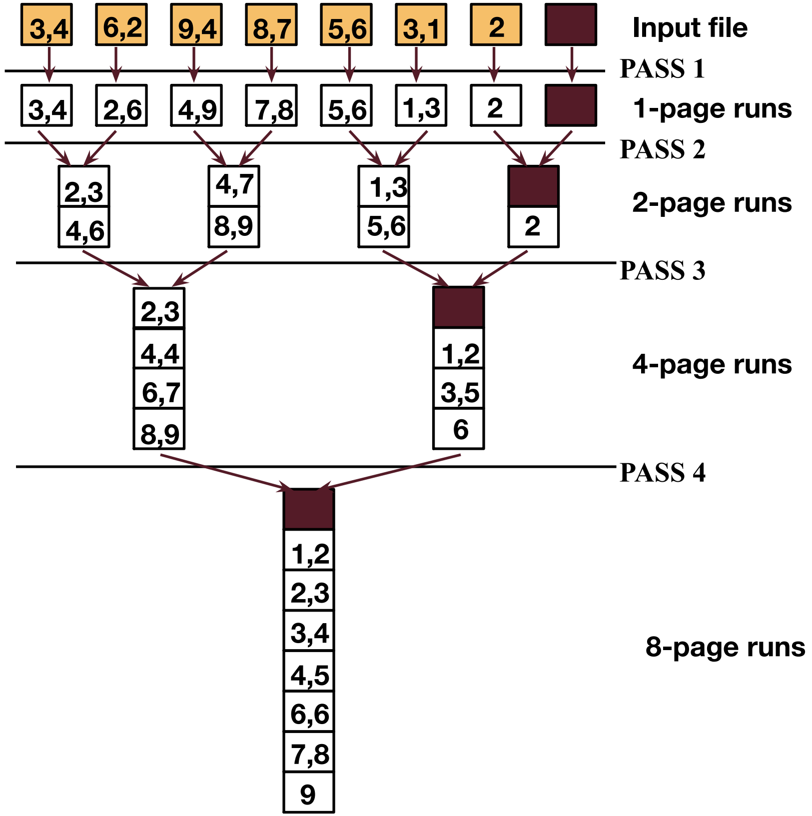


Sorting: 2-Way (a strawman), cont

- Pass 1 (conquer a batch):
  - read a page, sort it, write it.
  - only one buffer page is used
  - a repeated “batch job”
- Pass 2, 3, 4, …, etc. (merge via streaming):
  - requires 3 buffer pages
    - note: this has nothing to do with double buffering!
  - merge pairs of runs into runs twice as long
  - a streaming algorithm, as in the previous slide!
    - Drain/fill buffers as the data streams through them



Two-Way External Merge Sort

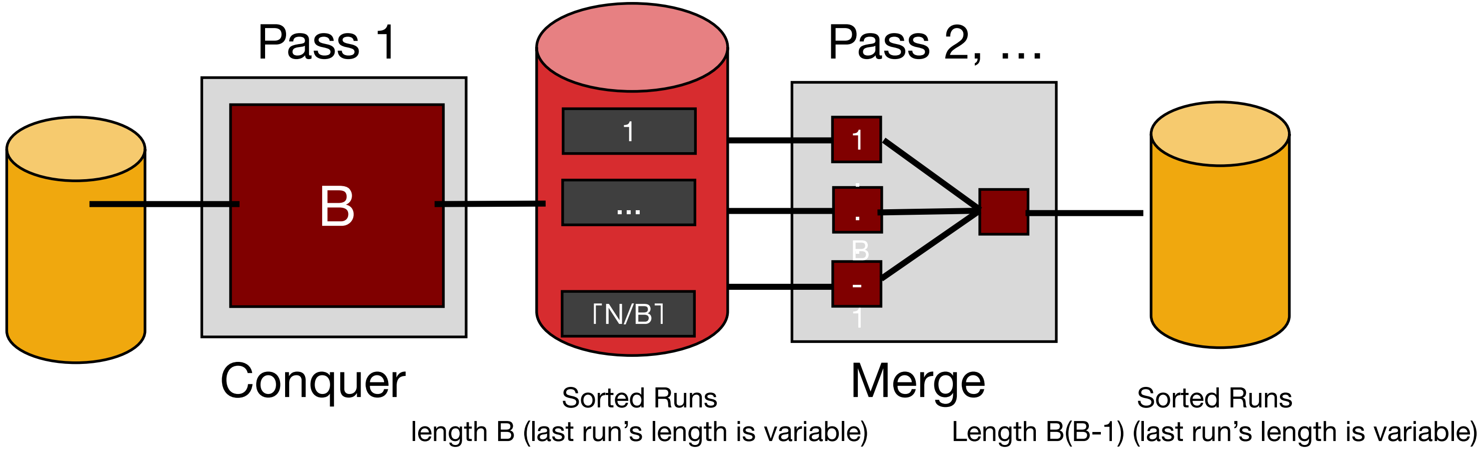


- Conquer and Merge:
  - sort subfiles and merge
- Each pass we read + write each page in file (2N)
- N pages in the file.
  - So, the number of passes is: $= \lceil \log_2 N \rceil + 1$
- So total cost is: $2N\left(\lceil \log_2 N \rceil + 1\right)$

General External Merge Sort

- We got more than 3 buffer pages. How can we utilize them?
  - Big batches in pass 1, many streams in merge passes

- To sort a file with N pages using B buffer pages:
  - Pass 1: use B buffer pages. Produce $\lceil N / B \rceil$ sorted runs of B pages each.
  - Pass 2, 3, …, etc.: merge B-1 runs at a time.



Cost of External Merge Sort

- Number of passes: $1 + \lceil \log_{B-1} \lceil N / B \rceil \rceil$
- Total I/Os = (I/Os per pass) * (# of passes) = 2*N * ($1 + \lceil \log_{B-1} \lceil N / B \rceil \rceil$)

- E.g., with 5 buffer pages, to sort 108 page file:
  - Pass 1: $\lceil 108 / 5 \rceil$ = 22 sorted runs of 5 pages each
    - last run is only 3 pages
  - Pass 2: $\lceil 22 / 4 \rceil$ = 6 sorted runs of 20 pages each
    - last run is only 8 pages
  - Pass 3: $\lceil 6 / 4 \rceil$ 2 sorted runs, 80 pages and 28 pages
  - Pass 4: Sorted file of 108 pages

Formula check: $1 + \lceil \log_4 22 \rceil = 1+3$ □ <u>4 passes</u> √

of Passes of External Sort

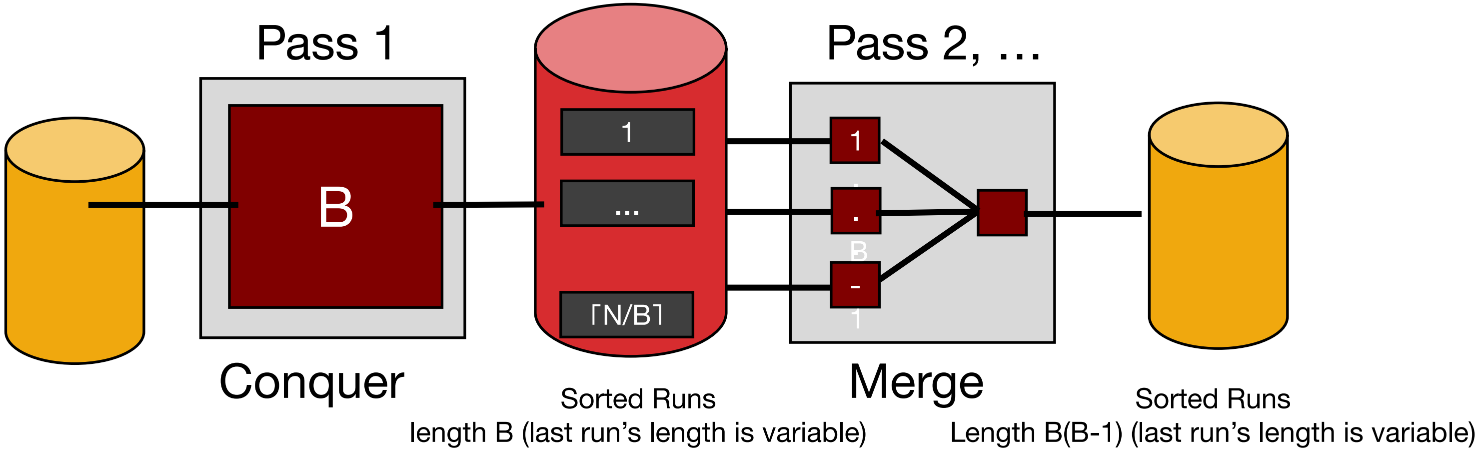
(Total I/O is 2N * # of passes)

N	B=3	B=5	B=9	B=17	B=129	B=257
100	7	4	3	2	1	1
1,000	10	5	4	3	2	2
10,000	13	7	5	4	2	2
100,000	17	9	6	5	3	3
1,000,000	20	10	7	5	3	3
10,000,000	23	12	8	6	4	3
100,000,000	26	14	9	7	4	4
1,000,000,000	30	15	10	8	5	4

Few runs can already sort large amounts of data!

Memory Requirement for External Sorting

- How big of a table can we sort in exactly two passes?
  - Each "sorted run" after pass 1 is of size B
  - Can merge up to B-1 sorted runs in pass 2
- Answer: B(B-1) ~ B^2 data in two passes, using B space
  - Sort X amount of data in ~B = sqrt(X) space (if we run only two passes)



Alternative: Hashing

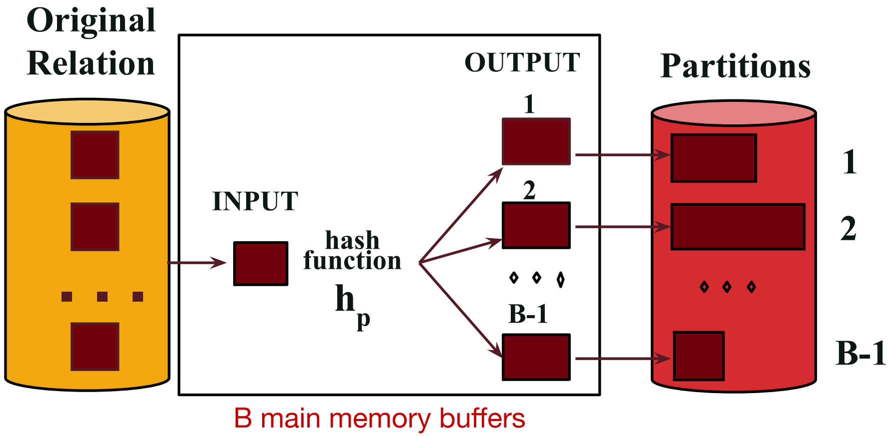
- Idea:
  - Many times we don't require order
    - E.g., remove duplicates, form groups
- Often just need to rendezvous matches
- Hashing does this
  - But how to do it out-of-core??

Divide

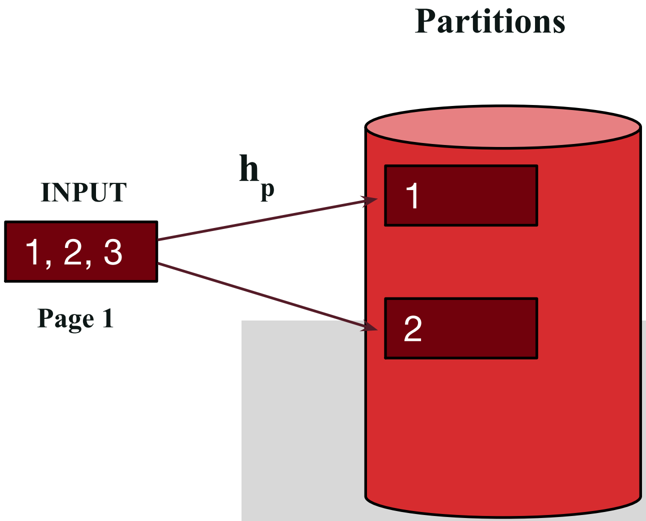
- Streaming Partition (divide):
  Use a hash function $\mathbf{h_p}$ to stream records to disk partitions
  - All matches rendezvous in the same partition.
  - Each partition a mix of values
  - Streaming algorithm to create partitions on disk:
    - "Spill" partitions to disk via output buffers

Two Phases: Divide

- Partition: (Divide)

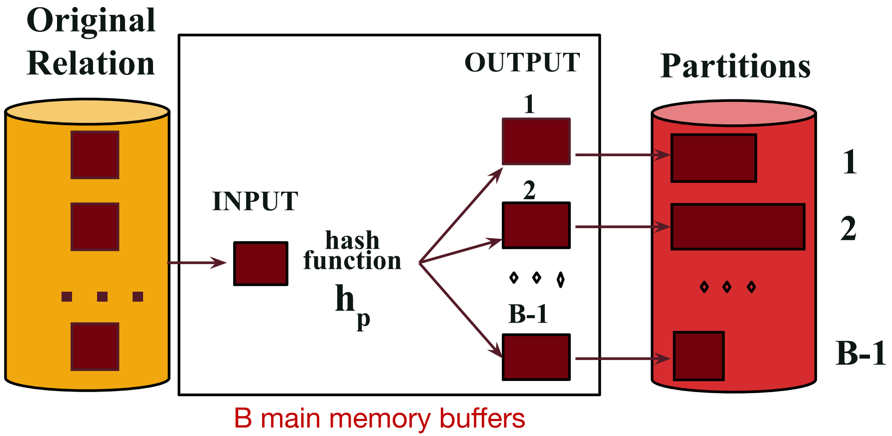


Example

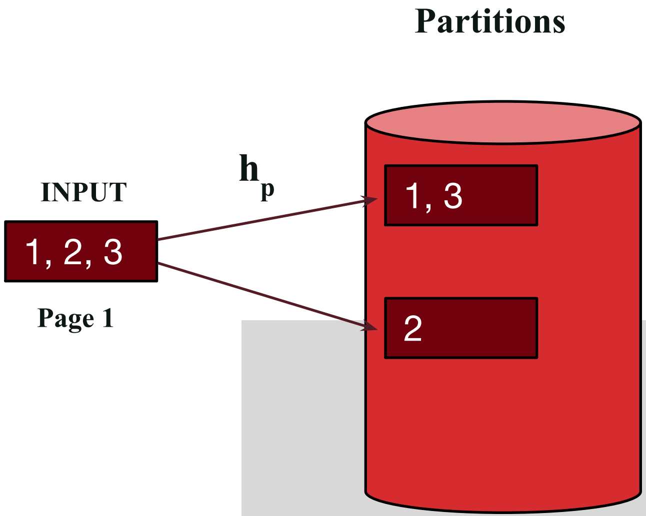


Two Phases: Divide

- Partition:
  (Divide)

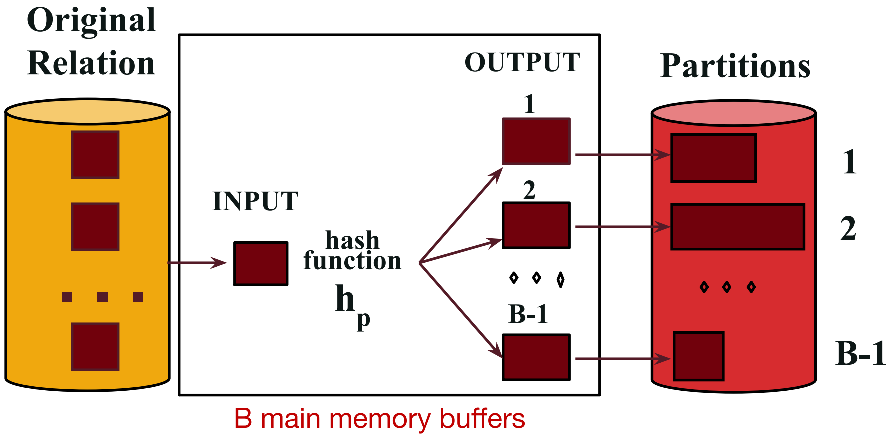


Example

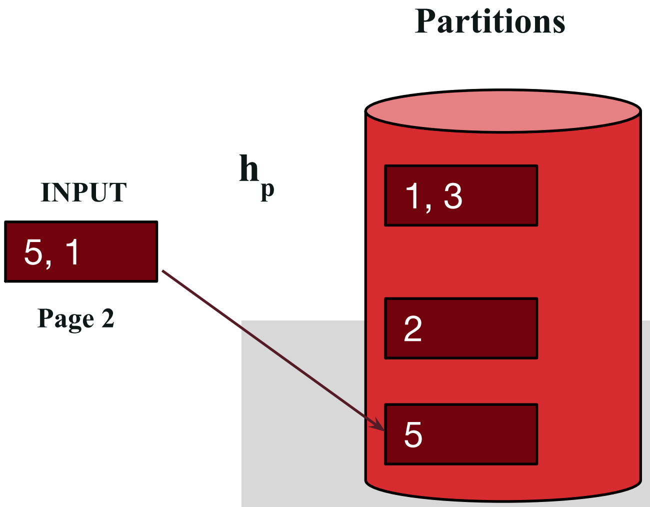


Two Phases: Divide

- Partition:
  (Divide)

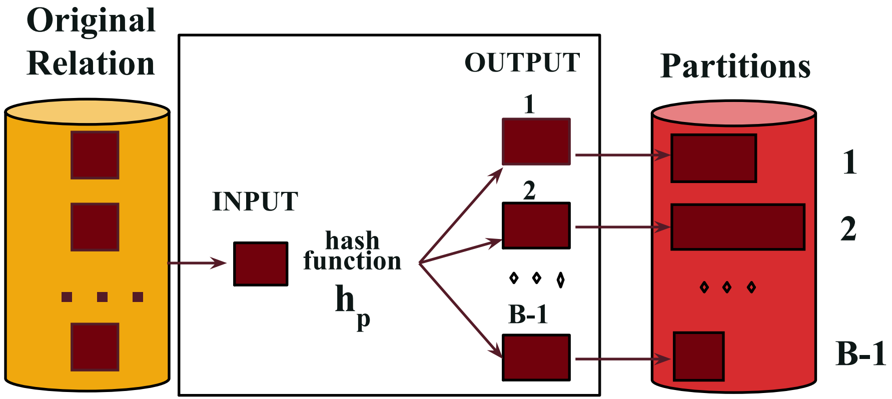


Example

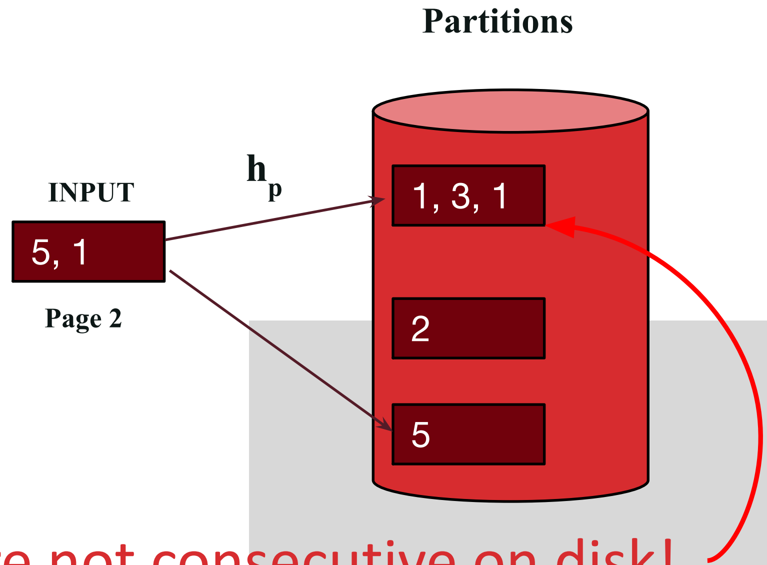


Two Phases: Divide

- Partition:
  (Divide)



Example



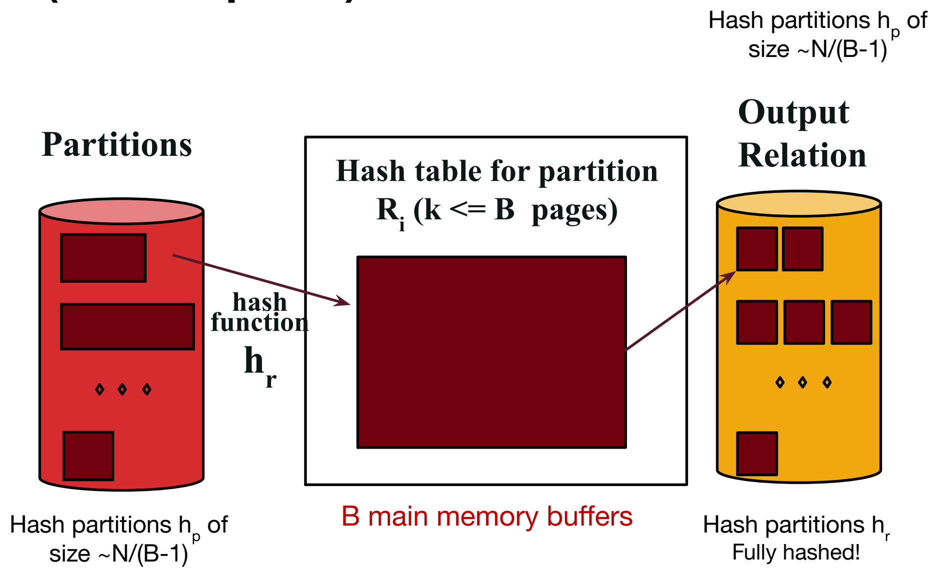
The 1's are not consecutive on disk!

Conquer

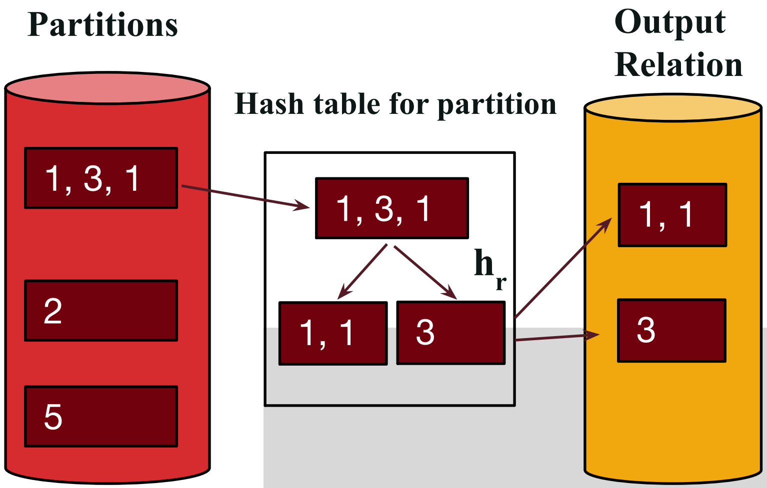
- ReHash (conquer):
  Read partitions into RAM hash table one at a time, using *different* hash function $\mathbf{h_r}$
  - Each bucket contains a small number of distinct values
- Then read out the RAM hash table buckets and write to disk
  - Ensuring that duplicate values are contiguous

Two Phases: Conquer

- Rehash:
  (Conquer)

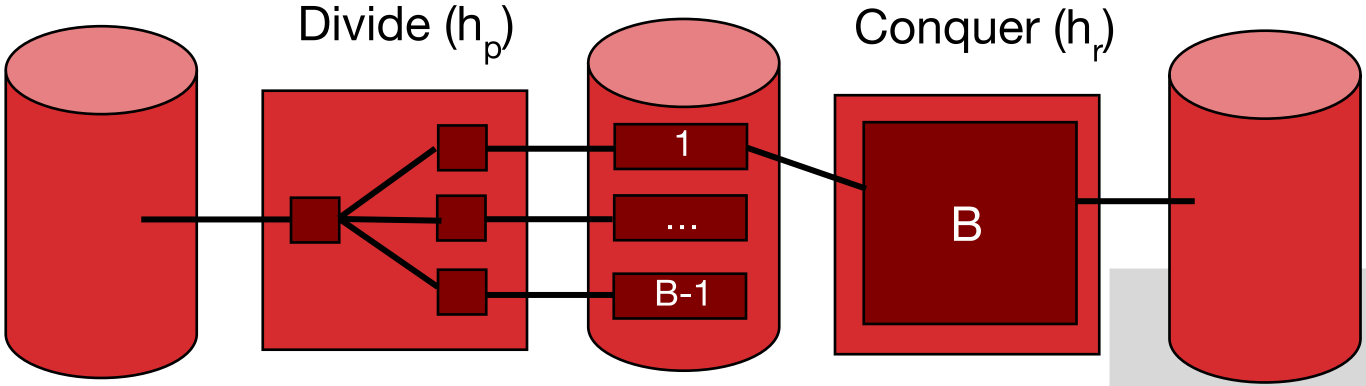


Example



Cost of External Hashing

Total I/Os ~ 2*N*(# passes) = 4*N
(includes initial read, final write)

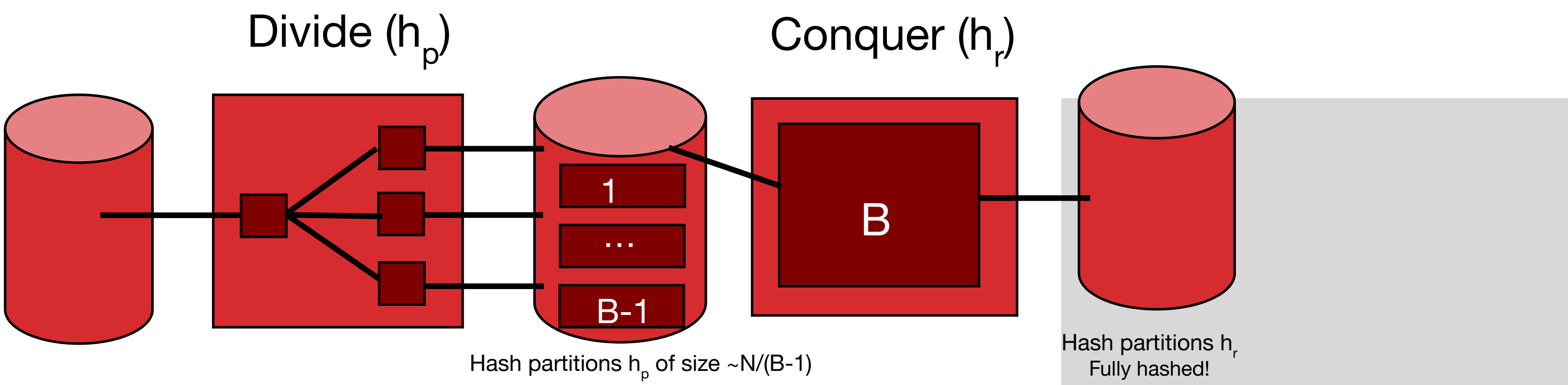


Hash partitions h_p of size ~N/(B-1)

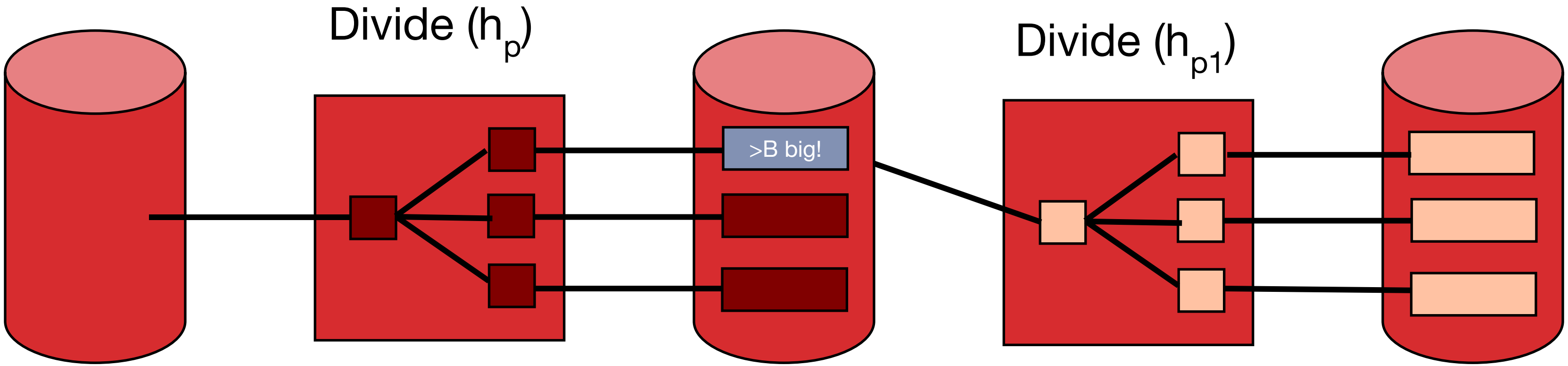
Hash partitions h_r
Fully hashed!

Memory Requirement

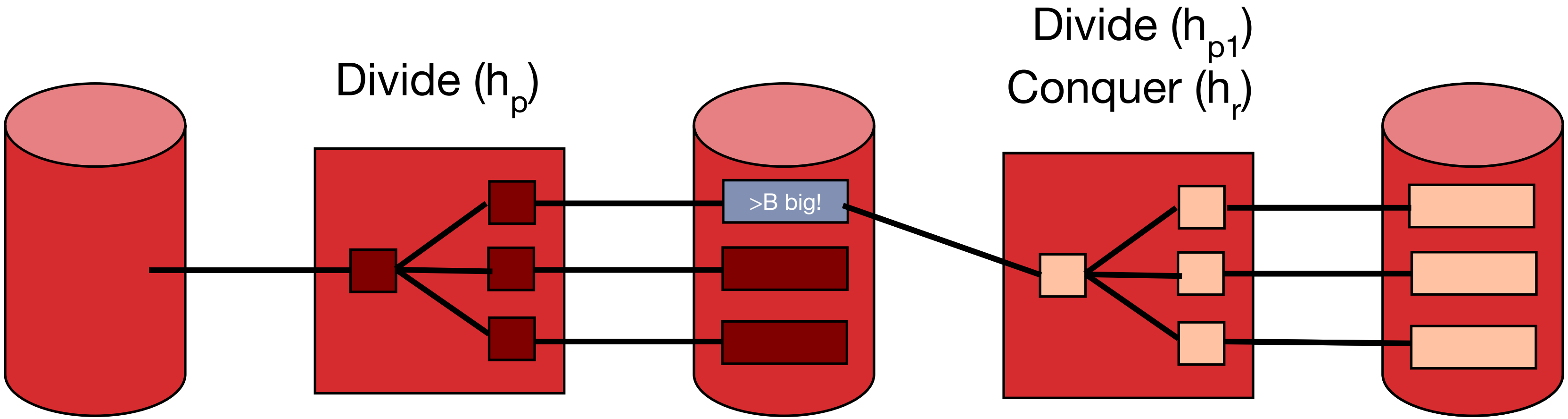
- How big of a table can we hash in exactly two passes?
  - B-1 “partitions” result from Pass 1
  - Each should be no more than B pages in size
  - Answer: B(B-1) ~ B^2
    - We can hash a table of size X in about $B = \sqrt{X}$ space (if we run only 2 passes)
  - Note: assumes hash function distributes records evenly!
- Have a bigger table? Recursive partitioning!



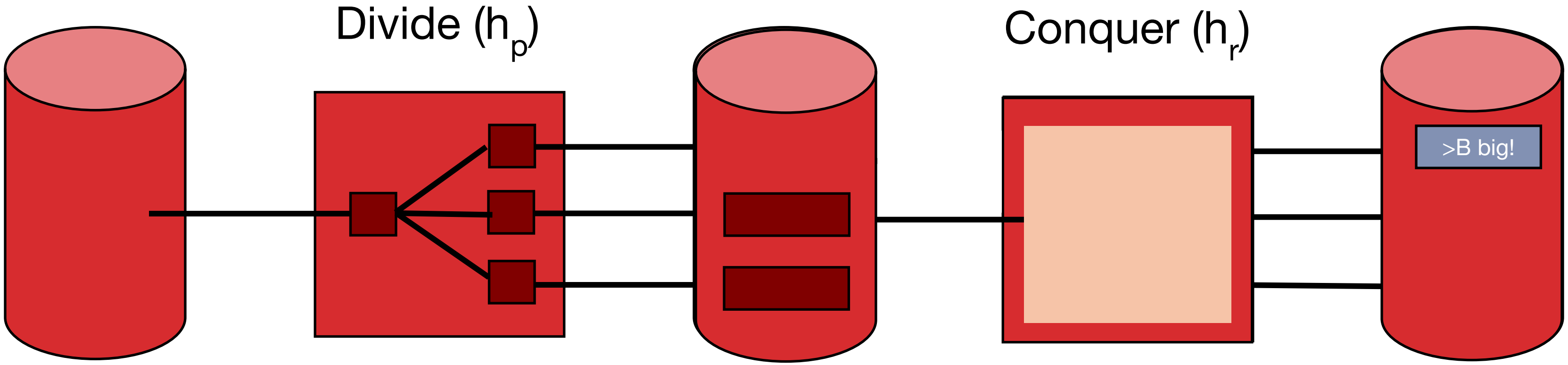
Recursive Partitioning, Pt 1



Recursive Partitioning, Pt 2

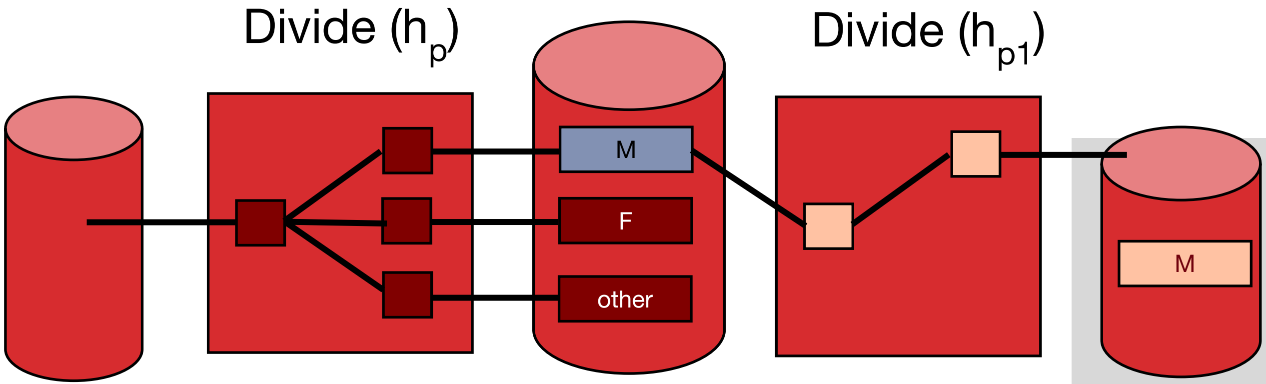


Recursive Partitioning, Pt 3



A Wrinkle: Duplicates

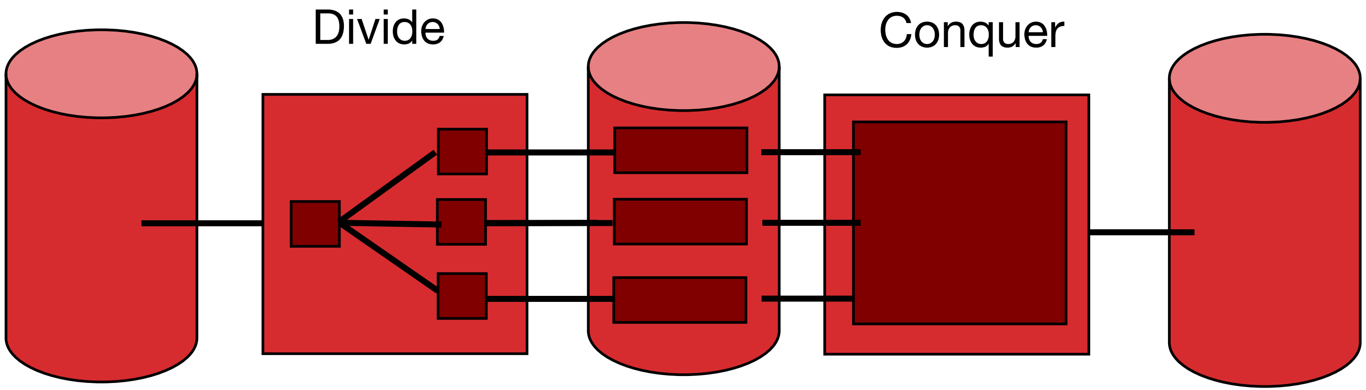
- Consider a dataset with a *very* frequent key
  - E.g., in a big table, consider the *gender* column
- What happens during recursive partitioning?



Question…

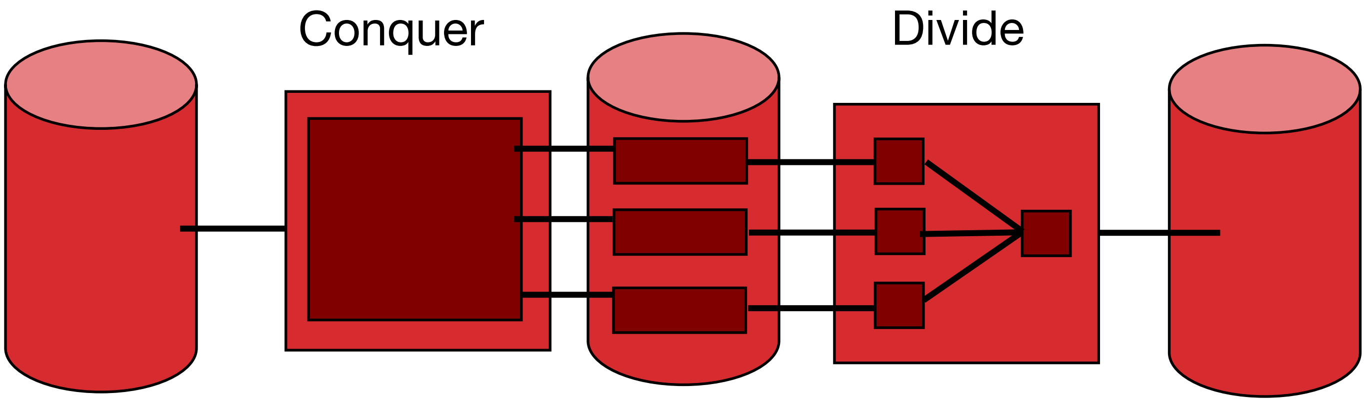
How does external hashing compare with external sorting?

Cost of External Hashing



cost ~ 4*N I/Os
(including initial read, final write)
(case for 1 divide pass and 1 conquer pass only. Not the general formula!)

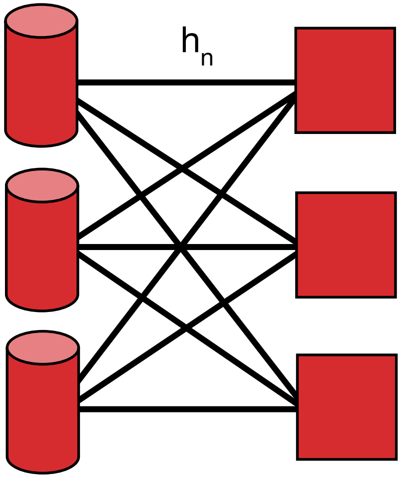
Cost of External Sorting



cost = 4*N I/Os
(including initial read, final write)
(case for 1 divide pass and 1 conquer pass only. Not the general formula!)

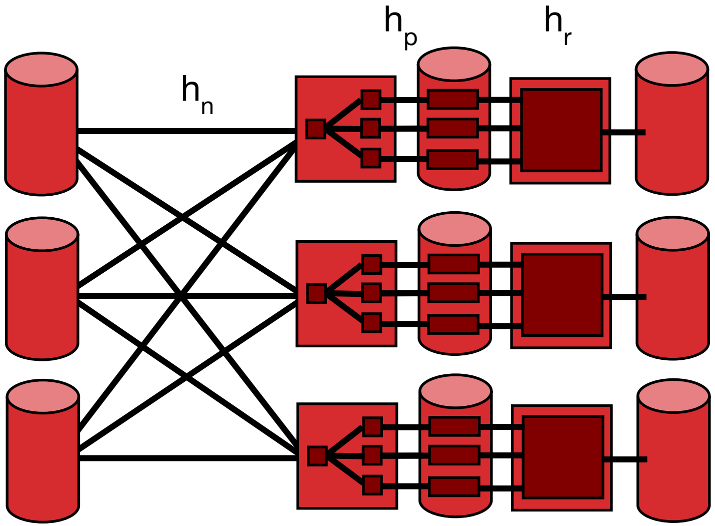
Parallelize me! Hashing Phase 1

- Phase 1: shuffle data across machines (hn)
  - streaming out to network as it is scanned
  - which machine for this record?
    use (yet another) independent hash function hn



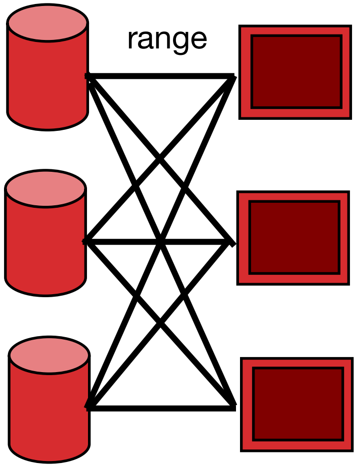
Parallelize me! Hashing Phase 2

- Phase 1: shuffle data across machines (hn)
- Receivers proceed with phase 1as data streams in
  - from local disk and network



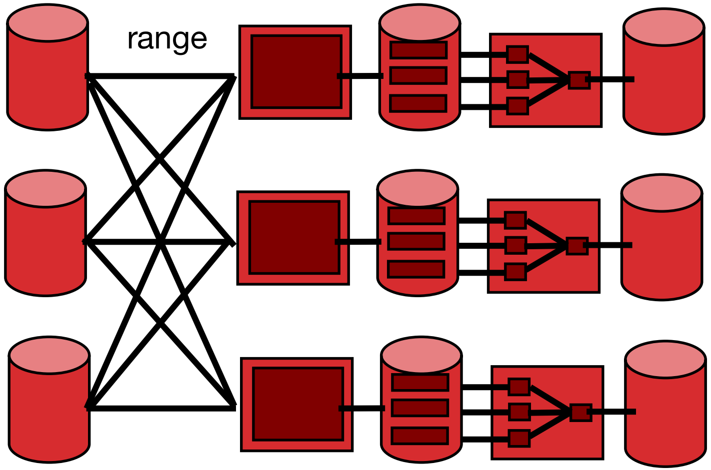
Parallelize me! Sorting

- Pass 1: shuffle data across machines
  - streaming out to network as it is scanned
  - which machine for this record?
    Split on value range (e.g. [-∞,10], [11,100], [101, ∞]).



Parallelize me! Sorting, cont

- Pass 1: shuffle data across machines
- Receivers proceed with pass 1 as the data streams in
- A Wrinkle: How to ensure ranges are the same #pages?!
  - i.e. avoid data skew?



So which is better ??

- Simplest analysis:
  - Same memory requirement for 2 passes
  - Same I/O cost
  - But we can dig a bit deeper…

Sorting vs Hashing

- Hashing pros:
  - For duplicate elimination, scales with # of values
    - Delete dups in first pass while partitioning on hp
    - Vs. sort which scales with # of items!
  - Easy to shuffle equally in parallel case
- Sorting pros:
  - Great if we need output to be sorted anyway
  - Not sensitive to duplicates or “bad” hash functions

Summary

- Sort/Hash Duality
  - Hashing is Divide & Conquer
  - Sorting is Conquer & Merge
- Sorting is overkill for rendezvous
  - But sometimes a win anyhow
- Don’t forget one pass streaming and double buffering
  - Can “hide” the latency of I/O behind CPU work