CS 188
Spring 2024

Regular Discussion 4 Solutions

Q1. Propositional Logic

(a) Provide justification for whether each of the following are correct or incorrect.

(i) $(X \vee Y) \models Y$

Incorrect: $(X \vee Y) \models Y$ if and only if $(X \vee Y) \wedge \neg Y$ is unsatisfiable; however, the latter is satisfied by $X = true$ and $Y = false$.

(ii) $\neg X \vee (Y \wedge Z) \models (X \implies Y)$

Correct: Via the same reasoning as the previous part, we can attempt to show that $(\neg X \vee (Y \wedge Z)) \wedge \neg(X \implies Y)$ is unsatisfiable as follows:

- $(\neg X \vee (Y \wedge Z)) \wedge \neg(X \implies Y)$
- $(\neg X \vee (Y \wedge Z)) \wedge \neg(\neg X \vee Y)$
- $(\neg X \vee (Y \wedge Z)) \wedge (X \wedge \neg Y)$

Its clear that for the RHS to evaluate to true, $X = true$ and $Y = false$. However, setting that automatically makes the LHS evaluate to false. Thus, the whole thing is unsatisfiable, so the original must be correct.

(iii) $(X \vee Y) \wedge (Z \vee \neg Y) \models (X \vee Z)$

Correct: In general, $A \models B$ if and only if $A \implies B$ is valid. To show that this works for this problem, we can write it as an implication and prove by counterexample that the statement is always valid. Consider $(X \vee Y) \wedge (Z \vee \neg Y) \implies (X \vee Z)$. In general, $A \implies B$ only evaluates to false if $A = true$ and $B = false$. In order for the RHS to evaluate to false, $X = false$ and $Z = false$. Plugging those in, the LHS evaluates to $(false \vee Y) \wedge (false \vee \neg Y)$, which can never evaluate to true. Therefore, that case never holds, so the implication is valid.

(b) Consider the following sentence:

$$[(Food \implies Party) \vee (Drinks \implies Party)] \implies [(Food \wedge Drinks) \implies Party] \; .$$

(i) Determine, using enumeration, whether this sentence is valid, satisfiable (but not valid), or unsatisfiable.

A simple truth table has eight rows, and shows that the sentence is true for all models and hence valid.

(ii) Convert the left-hand and right-hand sides of the main implication into CNF.

For the left-hand side we have:

- $(Food \implies Party) \vee (Drinks \implies Party)$
- $(\neg Food \vee Party) \vee (\neg Drinks \vee Party)$
- $(\neg Food \vee Party \vee \neg Drinks \vee Party)$
- $(\neg Food \vee \neg Drinks \vee Party)$

For the right-hand side we have:

- $(Food \wedge Drinks) \implies Party$
- $\neg(Food \wedge Drinks) \vee Party$
- $(\neg Food \vee \neg Drinks) \vee Party$
- $(\neg Food \vee \neg Drinks \vee Party)$

(iii) What do you observe about the LHS and RHS after converting to CNF? Explain how your results prove the answer to part b.i.

The two sides are identical in CNF, and hence the original sentence is of the form $P \implies P$, which is valid for any P.

Q2. More Propositional Logic

(a) Pacman has lost the meanings for the symbols in his knowledge base! Luckily he still has the list of sentences in the KB and the English description he used to create his KB.

For each English sentence on the left, there is a corresponding logical sentence in the knowledge base on the right (not necessarily the one across from it). Your task is to recover this matching. Once you have, please fill in the blanks with the English sentence that matches each symbol.

English	**Knowledge Base**
There is a ghost at (0, 1).	$(C \vee B) \wedge (\neg C \vee \neg B)$
If Pacman is at (0, 1) and there is a ghost at (0, 1), then Pacman is not alive.	$C \wedge \neg D$
Pacman is at (0, 0) and there is no ghost at (0, 1).	$\neg A \vee \neg(B \wedge D)$
Pacman is at (0, 0) or (0, 1), but not both.	D

$A =$ ______________________ $C =$ ______________________

$B =$ ______________________ $D =$ ______________________

$A =$ Pacman is alive.

$B =$ Pacman is at (0, 1).

$C =$ Pacman is at (0, 0).

$D =$ There is a ghost at (0, 1).