

## 1 Probability

Use the probability table to calculate the following values:

$X_1$	$X_2$	$X_3$	$P(X_1, X_2, X_3)$
0	0	0	0.05
1	0	0	0.1
0	1	0	0.4
1	1	0	0.1
0	0	1	0.1
1	0	1	0.05
0	1	1	0.2
1	1	1	0.0

1.  $P(X_1 = 1, X_2 = 0)$

2.  $P(X_3 = 0)$

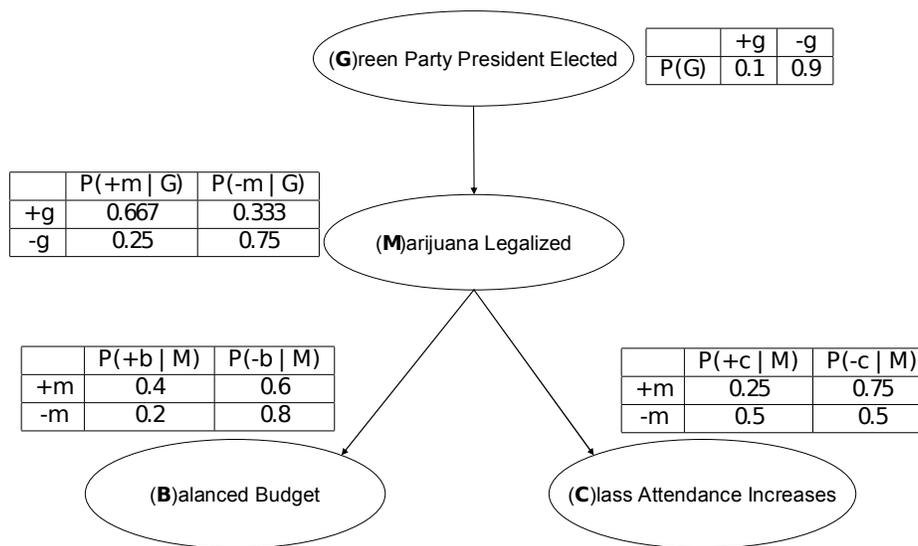
3.  $P(X_2 = 1|X_3 = 1)$

4.  $P(X_1 = 0|X_2 = 1, X_3 = 1)$

5.  $P(X_1 = 0, X_2 = 1|X_3 = 1)$

## Q2. Bayes Nets: Green Party President

In a parallel universe the Green Party is running for presidency. Whether a Green Party President is elected (G) will have an effect on whether marijuana is legalized (M), which then influences whether the budget is balanced (B), and whether class attendance increases (C). Armed with the power of probability, the analysts model the situation with the Bayes Net below.



1. The full joint distribution is given below. Fill in the missing values.

<i>G</i>	<i>M</i>	<i>B</i>	<i>C</i>	$P(G, M, B, C)$	<i>G</i>	<i>M</i>	<i>B</i>	<i>C</i>	$P(G, M, B, C)$
+	+	+	+	1/150	-	+	+	+	
+	+	+	-		-	+	+	-	27/400
+	+	-	+	1/100	-	+	-	+	
+	+	-	-		-	+	-	-	81/800
+	-	+	+	1/300	-	-	+	+	27/400
+	-	+	-	1/300	-	-	+	-	27/400
+	-	-	+		-	-	-	+	
+	-	-	-	1/75	-	-	-	-	27/100

2. Now, add a node *S* to the Bayes net that reflects the possibility that a new scientific study could influence the probability that marijuana is legalized. Assume that the study does not directly influence B or C. Draw the new Bayes net below. Which CPT or CPT's need to be modified?

### Q3. [Optional Logic Review] In What Worlds?

(a) We wish to come up with hypotheses that entail the following sentences:

- $S_1: X_1 \wedge X_2 \implies Y$
- $S_2: \neg X_1 \vee X_2 \implies Y$

In this problem, we want to come up with a hypothesis  $H$  such that  $H \models S_1 \wedge H \models S_2$ .

(i) Assume we have the hypothesis  $H: Y \iff X_1 \vee X_2$ .

Does  $H$  entail  $S_1$ ?     Yes     No

Does  $H$  entail  $S_2$ ?     Yes     No

(ii) Pretend that we have obtained a magical solver,  $SAT(s)$  which takes in a sentence  $s$  and returns *true* if  $s$  is satisfiable and *false* otherwise. We wish to use this solver to determine whether a hypothesis  $H'$  entails the two sentences  $S_1$  and  $S_2$ . Mark all of the following expressions that correctly return *true* if and only if  $H' \models S_1 \wedge H' \models S_2$ . If none of the expressions are correct, select "None of the above".

- $SAT(H' \wedge \neg(S_1 \wedge S_2))$         $SAT(\neg H' \vee (S_1 \wedge S_2))$
- $\neg SAT(H' \wedge \neg(S_1 \wedge S_2))$         $\neg SAT(\neg H' \vee (S_1 \wedge S_2))$
- None of the above

Four people, Alex, Betty, Cathy, and Dan are going to a family gathering. They can bring dishes or games. They have the following predicates in their vocabulary:

[topsep=-10pt] $Brought(p, i)$ : Person  $p$  brought a dish or game  $i$ .  $Cooked(p, d)$ : Person  $p$  cooked dish  $d$ .  $Played(p, g)$ : Person  $p$  played game  $g$ .

(b) Select which first-order logic sentences are syntactically correct translations for the following English sentences. You must use the syntax shown in class (eg.  $\forall, \exists, \wedge, \implies, \iff$ ). **Please select all that apply.**

(i) At least one dish cooked by Alex was brought by Betty.

- $\exists d Cooked(A, d) \wedge Brought(B, d)$
- $[\exists d Cooked(A, d)] \wedge [\forall d' \wedge (d' = d) Brought(B, d')]$
- $\neg[\forall d Cooked(A, d) \vee Brought(B, d)]$
- $\exists d_1, d_2 Cooked(A, d_1) \wedge (d_2 = d_1) \wedge Brought(B, d_2)$

(ii) At least one game played by Cathy is only played by people who brought dishes.

- $\neg[\forall g Played(C, g) \vee [\exists p Played(p, g) \implies \forall d Brought(p, d)]]$
- $\forall p \exists g Played(C, g) \wedge Played(p, g) \implies \exists d Brought(p, d)$
- $\exists g Played(C, g) \implies \forall p \exists d Played(p, g) \wedge Brought(p, d)$
- $\exists g Played(C, g) \wedge [\forall p Played(p, g) \implies \exists d, Brought(p, d)]$

(c) Assume we have the following sentence with variables  $A, B, C$ , and  $D$ , where each variable takes Boolean values:

$$S3 : (A \vee B \vee \neg C) \wedge (A \vee \neg B \vee D) \wedge (\neg B \vee \neg D)$$

(i) For the above sentence  $S3$ , state how many worlds make the sentence true. [Hint: you can do this and the next part without constructing a truth table!]

(ii) Does  $S3 \models (A \wedge B \wedge D)$ ?       Yes       No