Inference by Conditioning

(Chapter 8)

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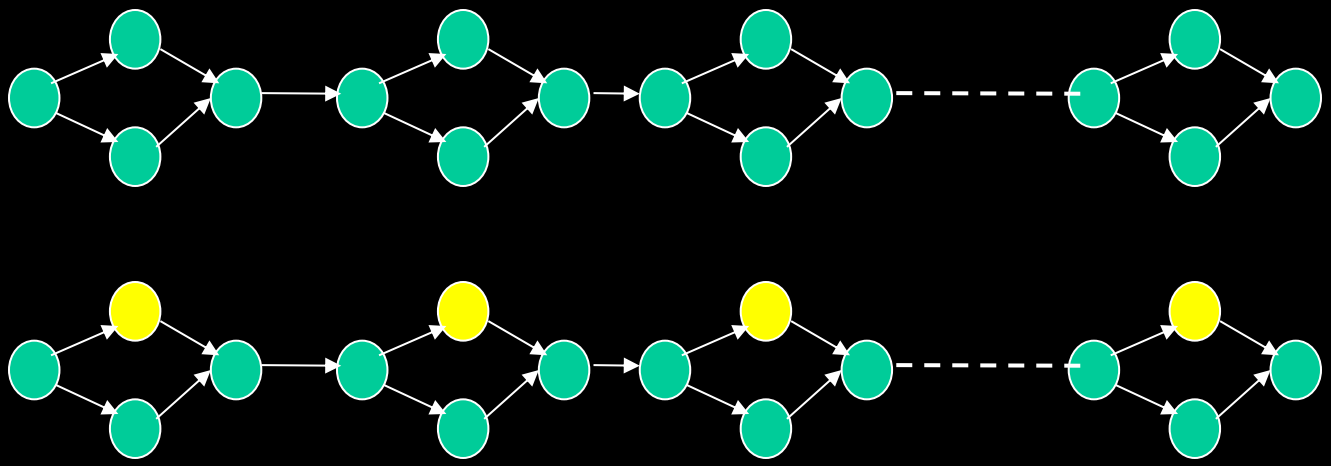
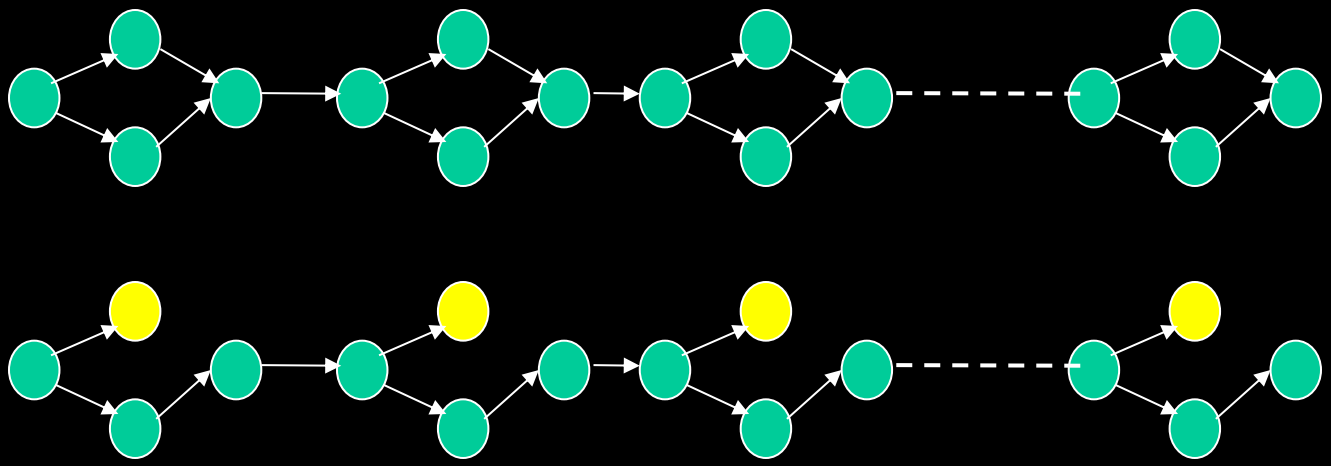
Cutset Conditioning



Cutset Conditioning



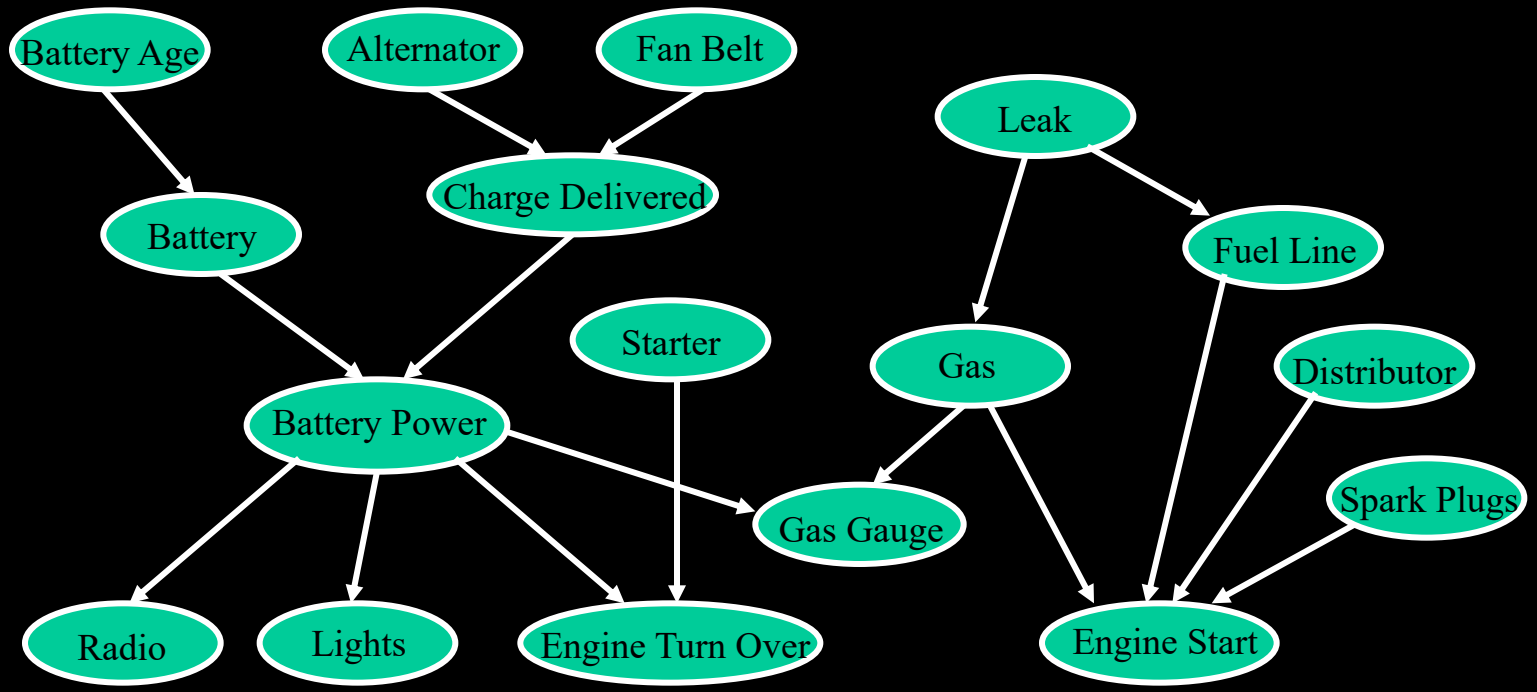
condition on enough nodes to get a polytree

time complexity: $O(n\ c^{S})$

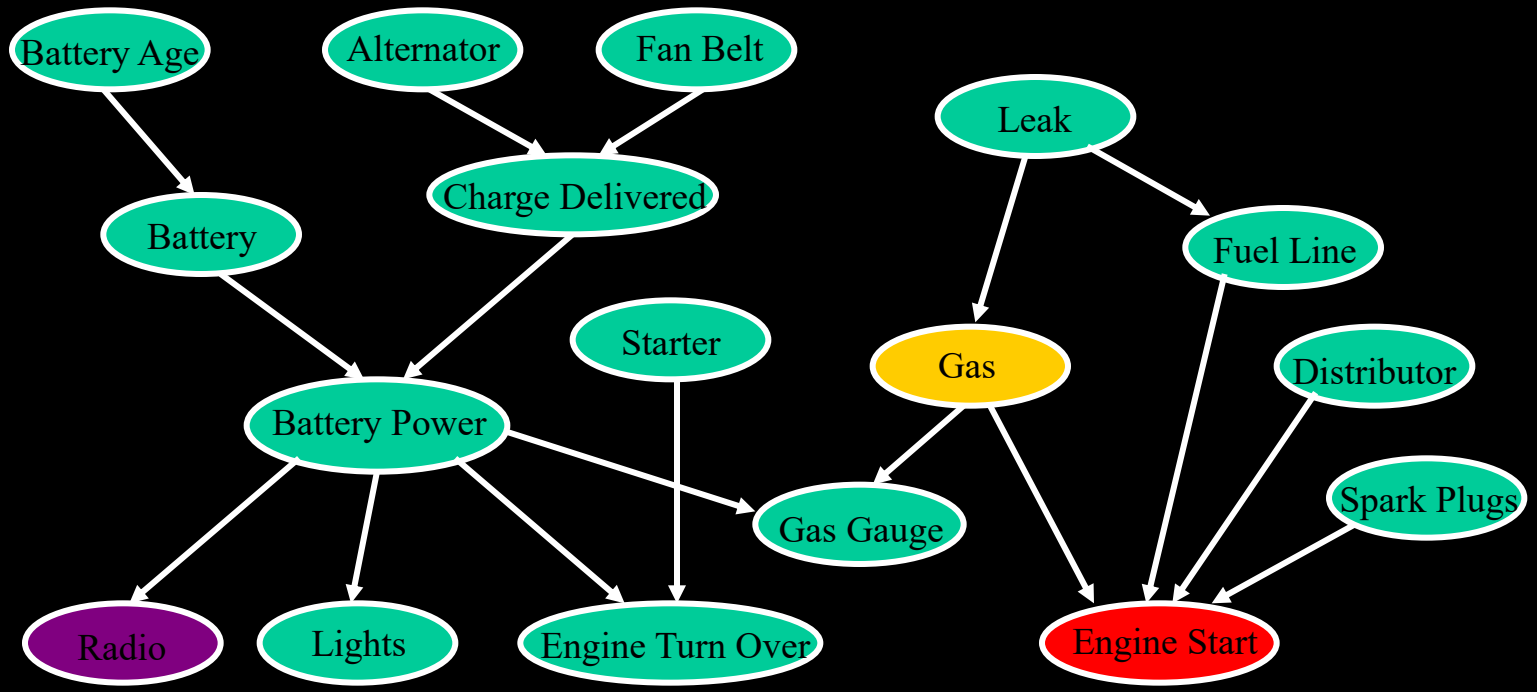
Recursive Conditioning

condition to decompose

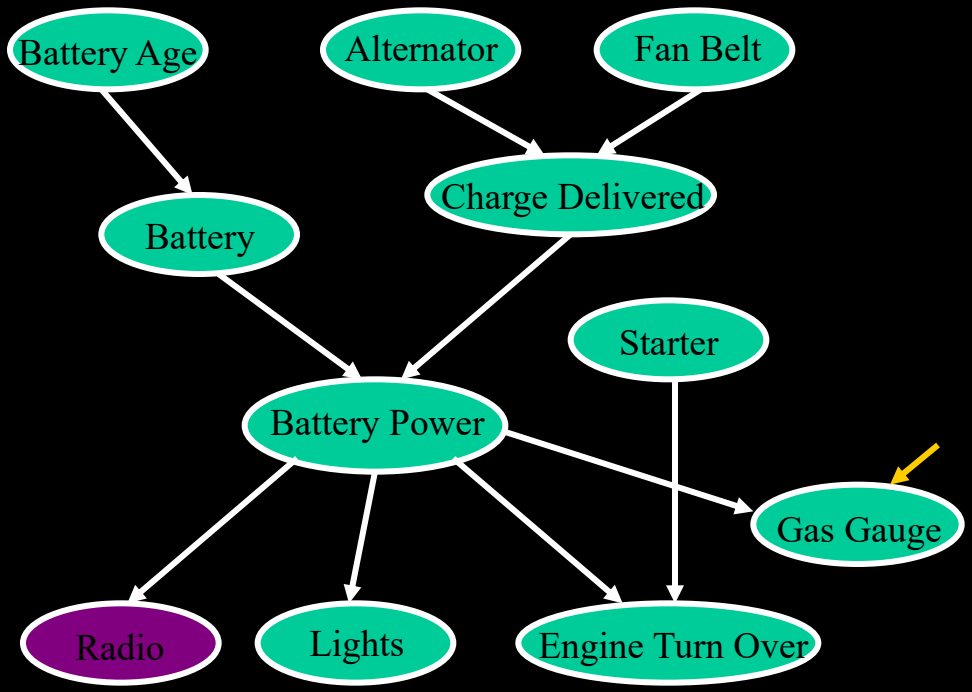
Decomposition

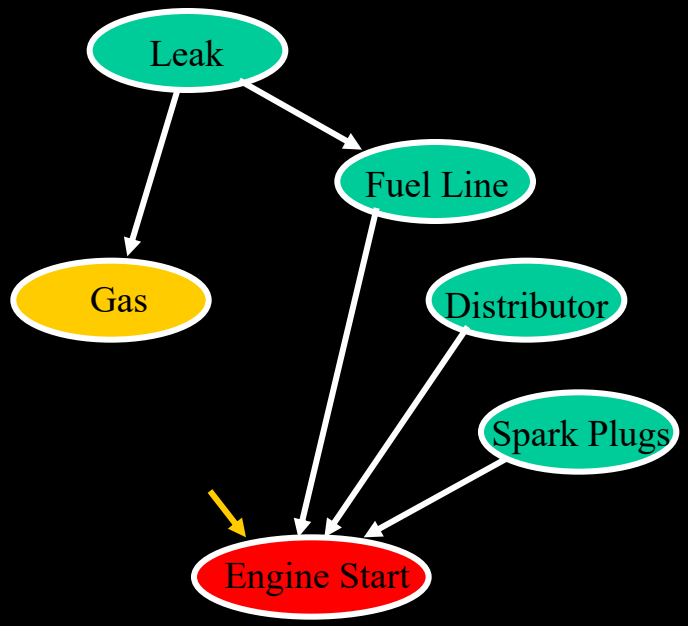


Decomposition

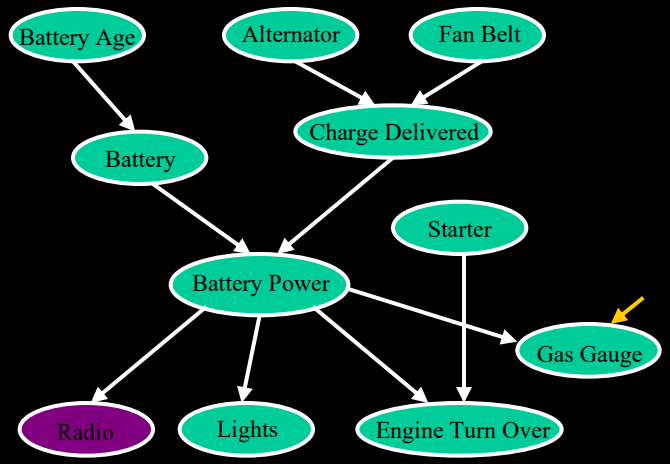


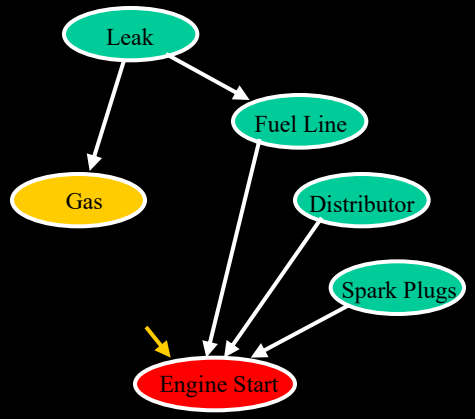
Decomposition





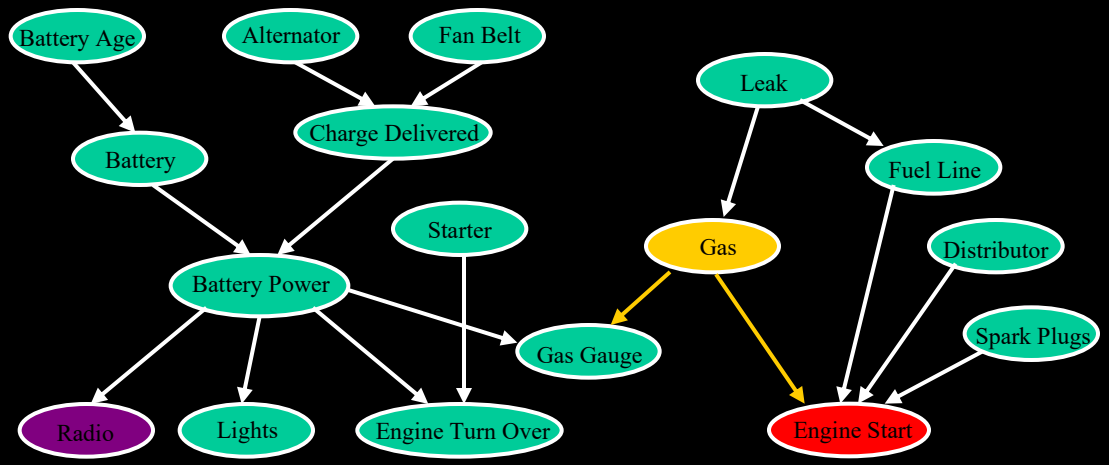
Decomposition





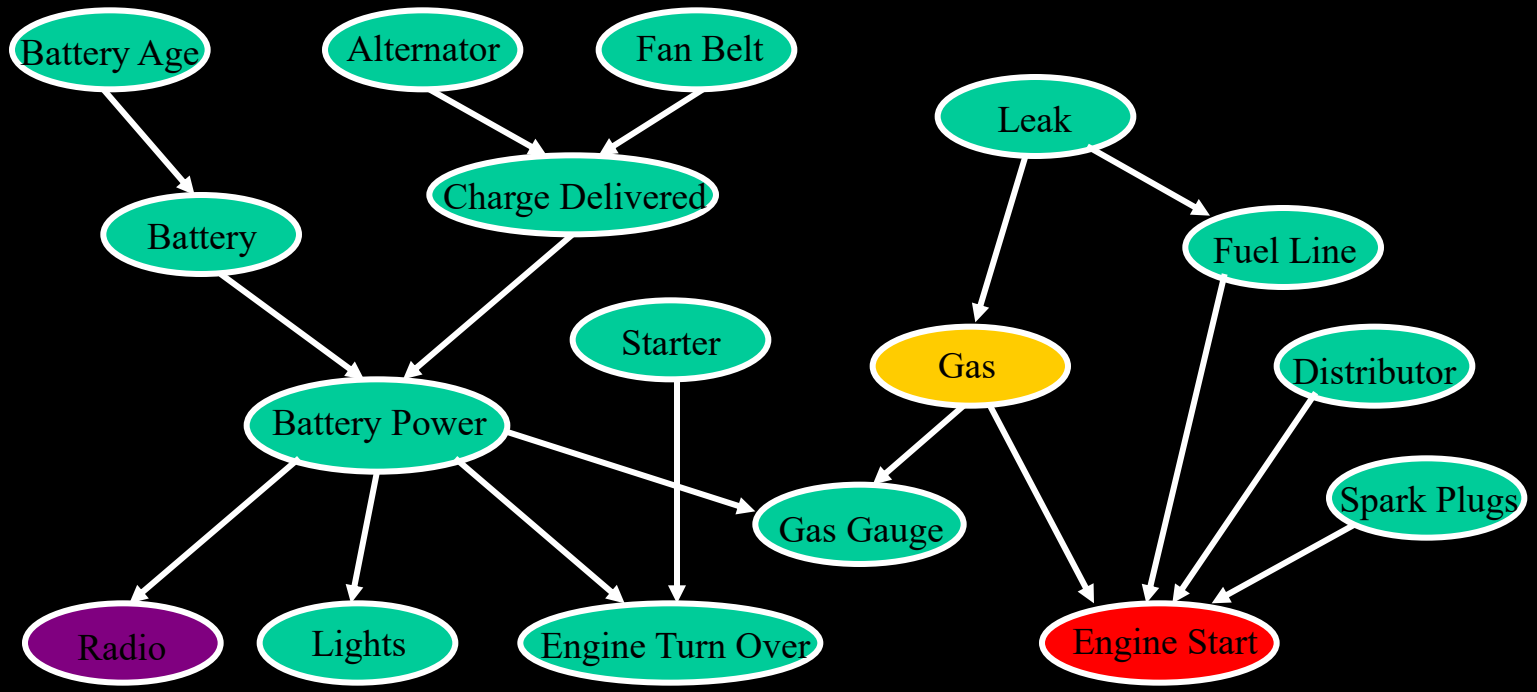
LP * RP

Decomposition

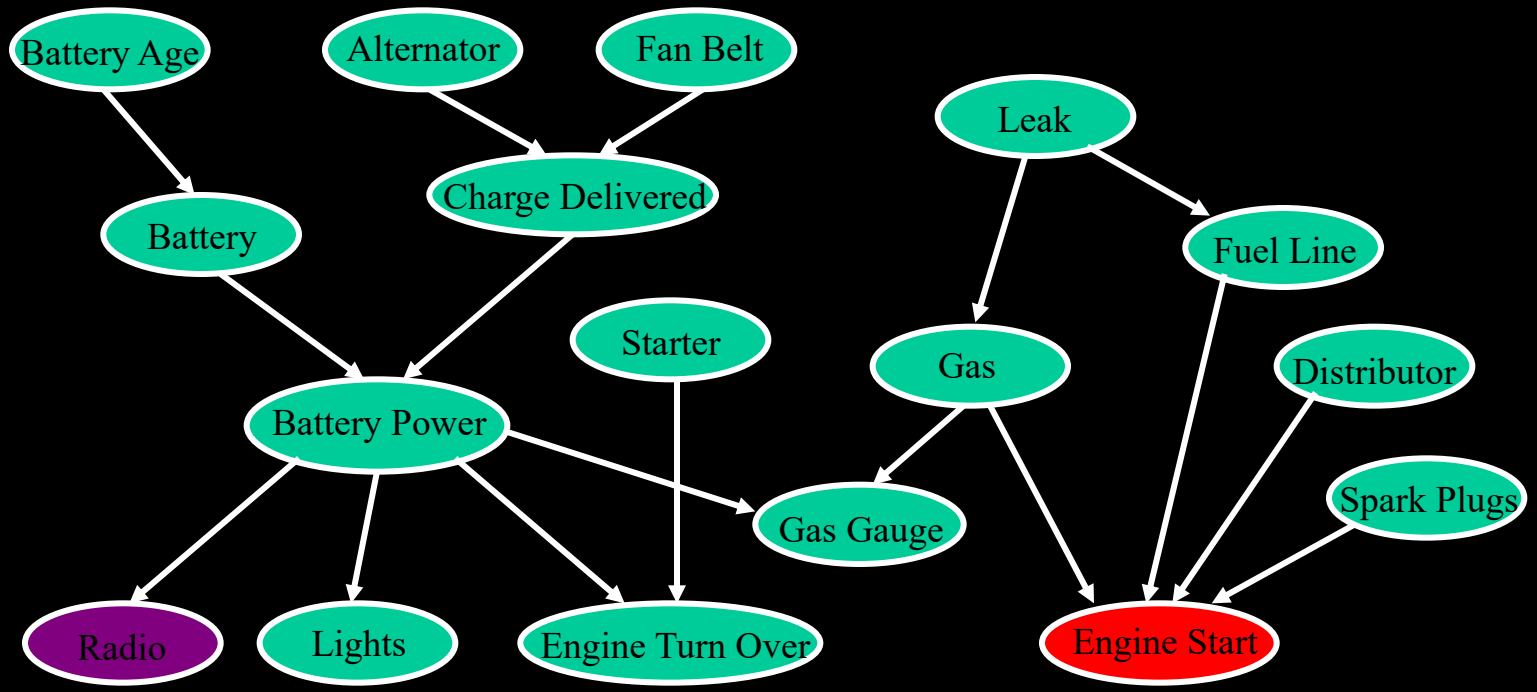


LP * RP

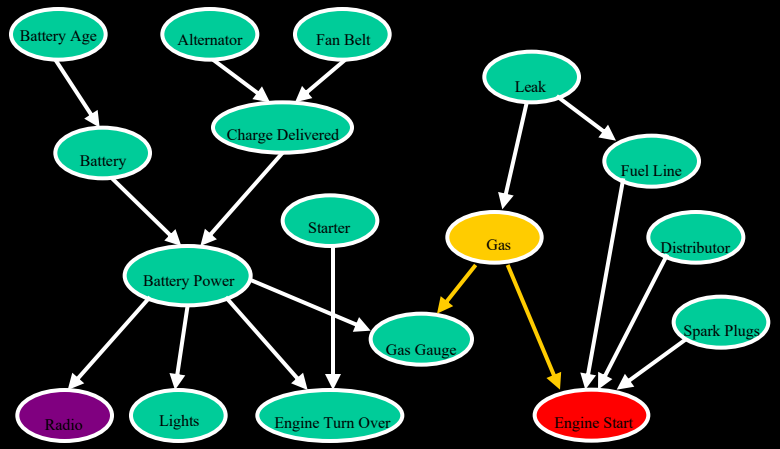
Causal Network



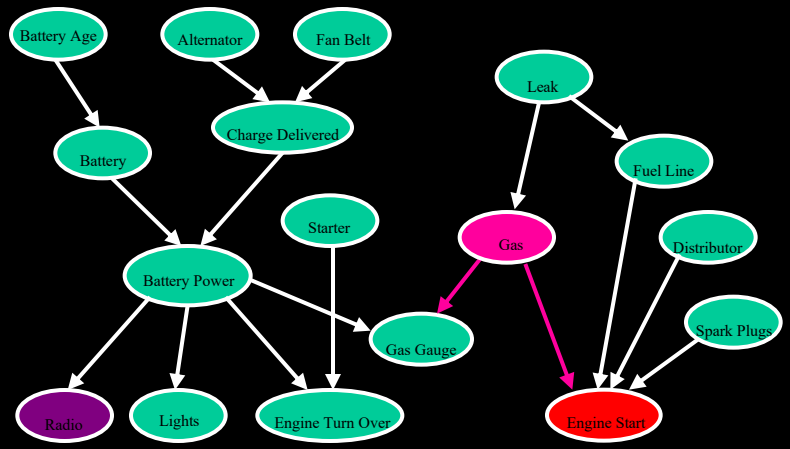
Causal Network



Case Analysis

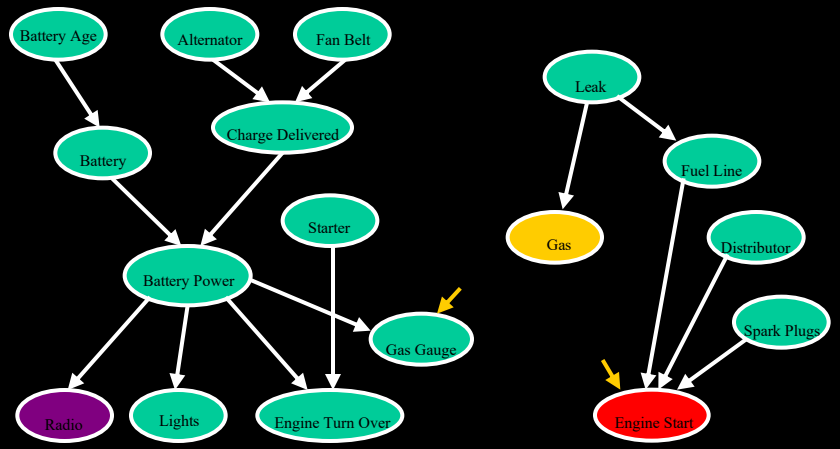


Case I

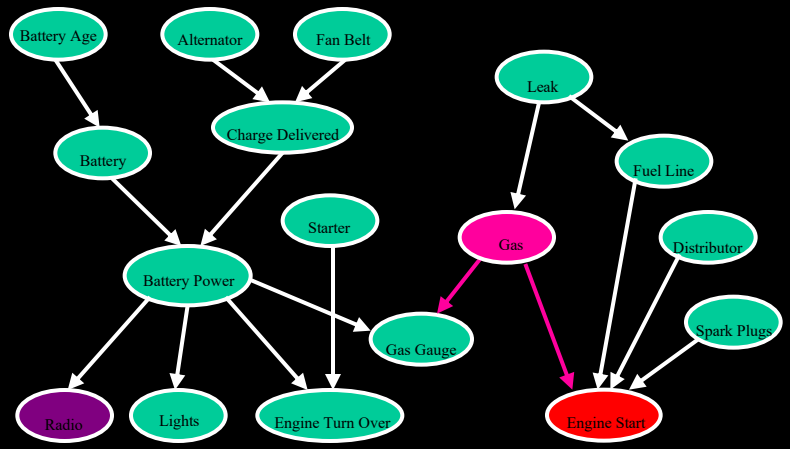


Case II

Case Analysis



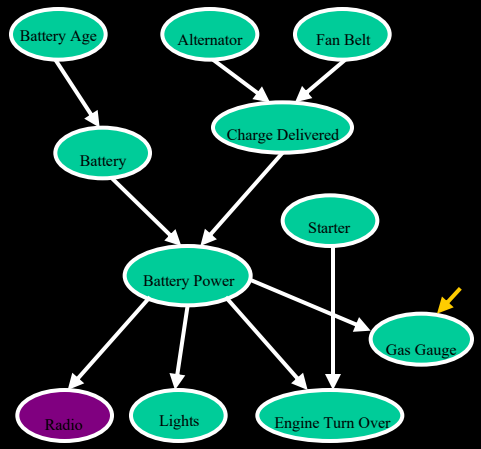
Case I

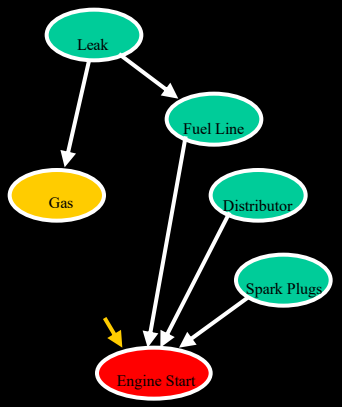


Case II

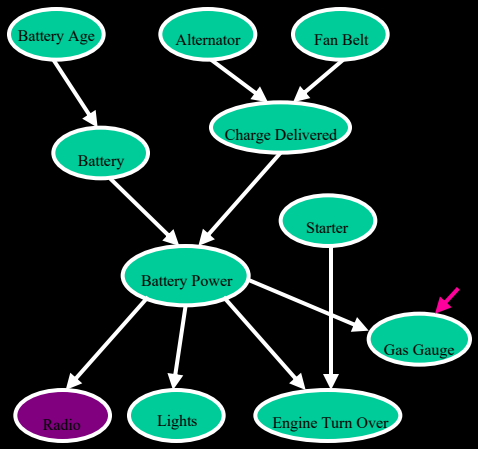
LP * RP

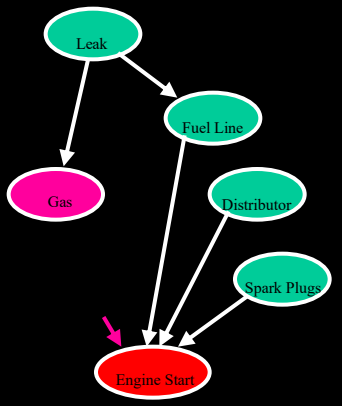
Case Analysis





Case I

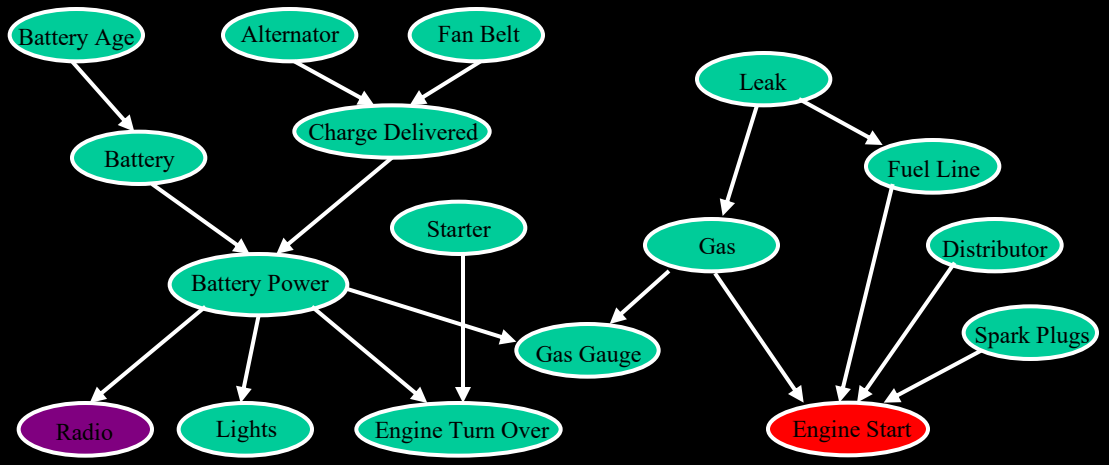




Case II

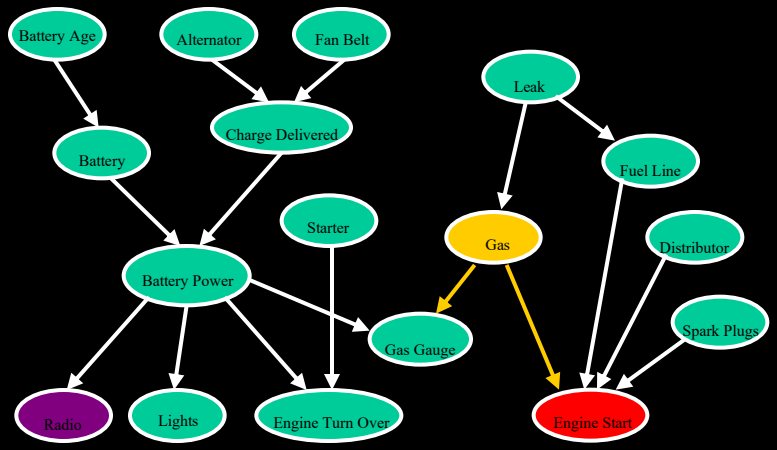
LP * RP + LP * RP

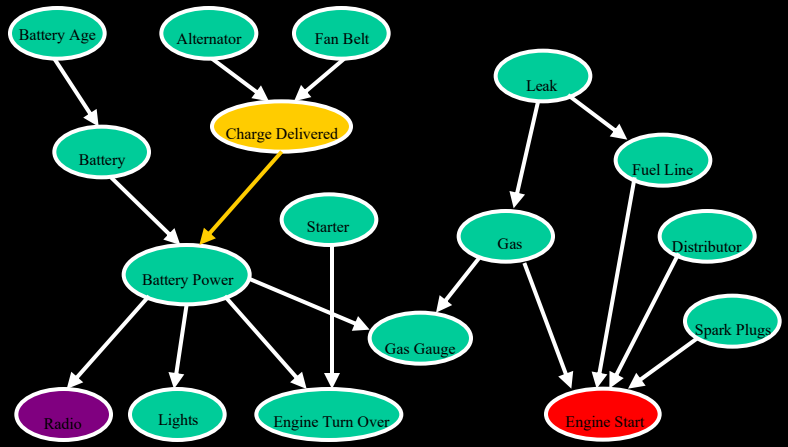
Case Analysis



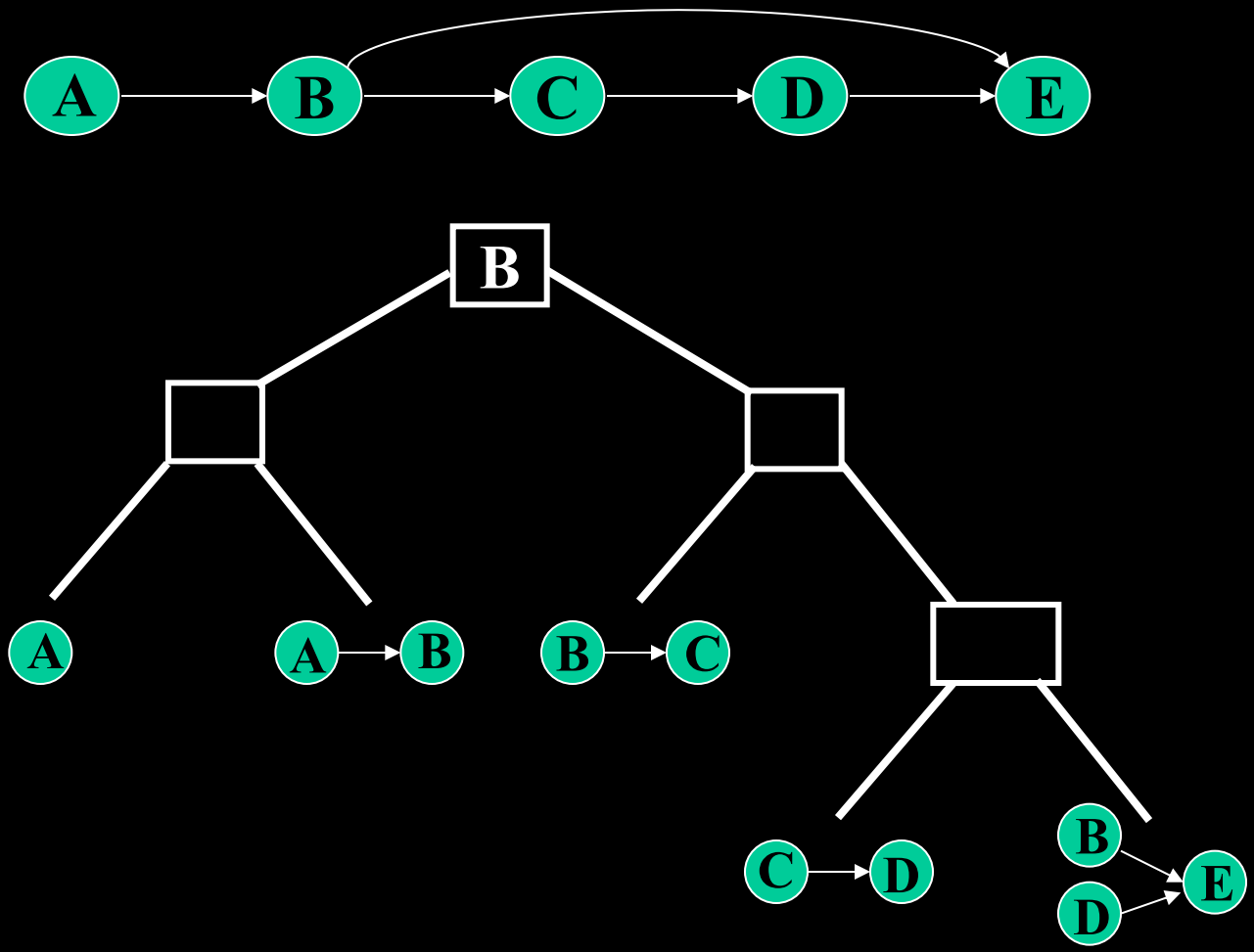
LP * RP + LP * RP

- **Decomposition and Case Analysis can answer any query**
- **Non-Deterministic!**

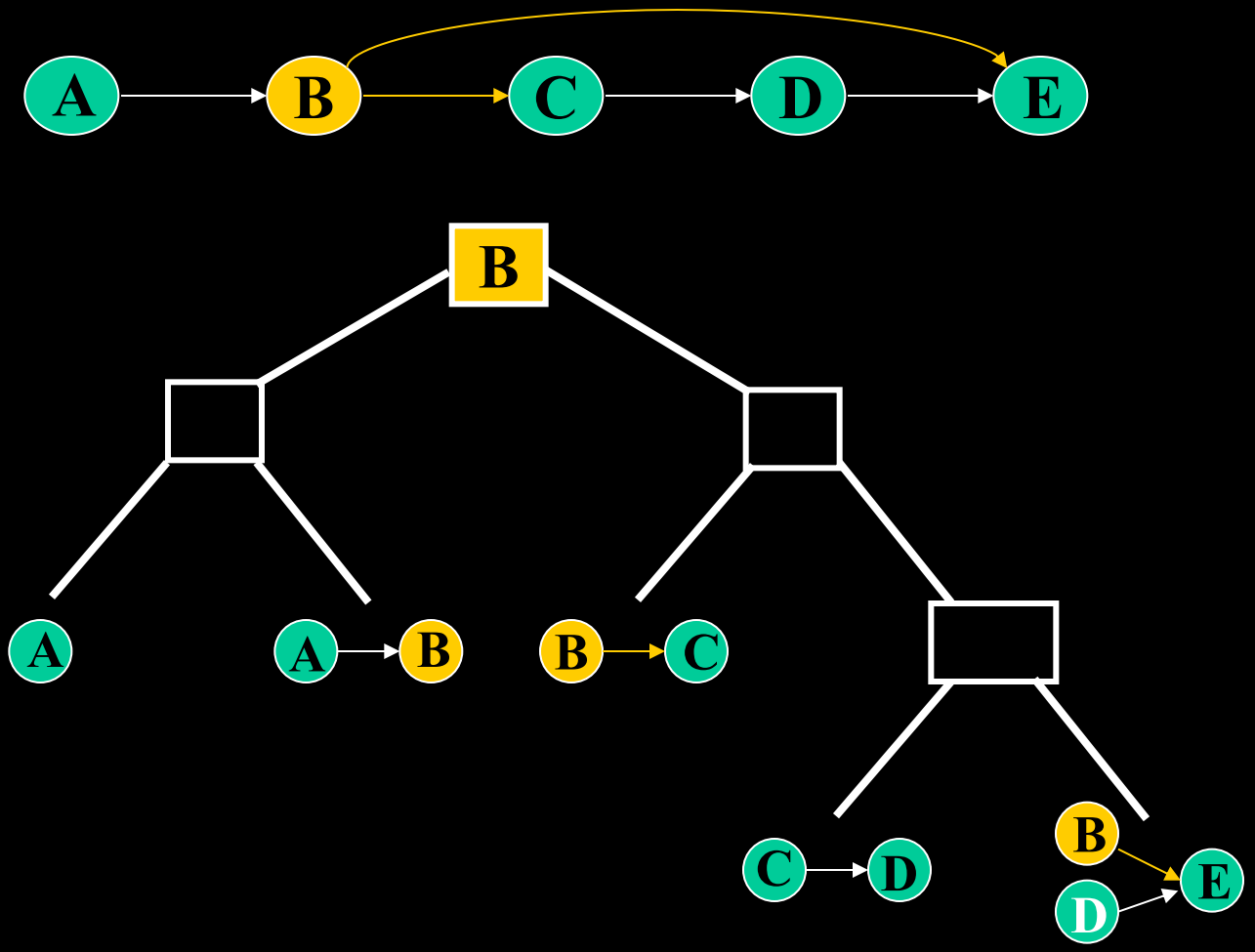




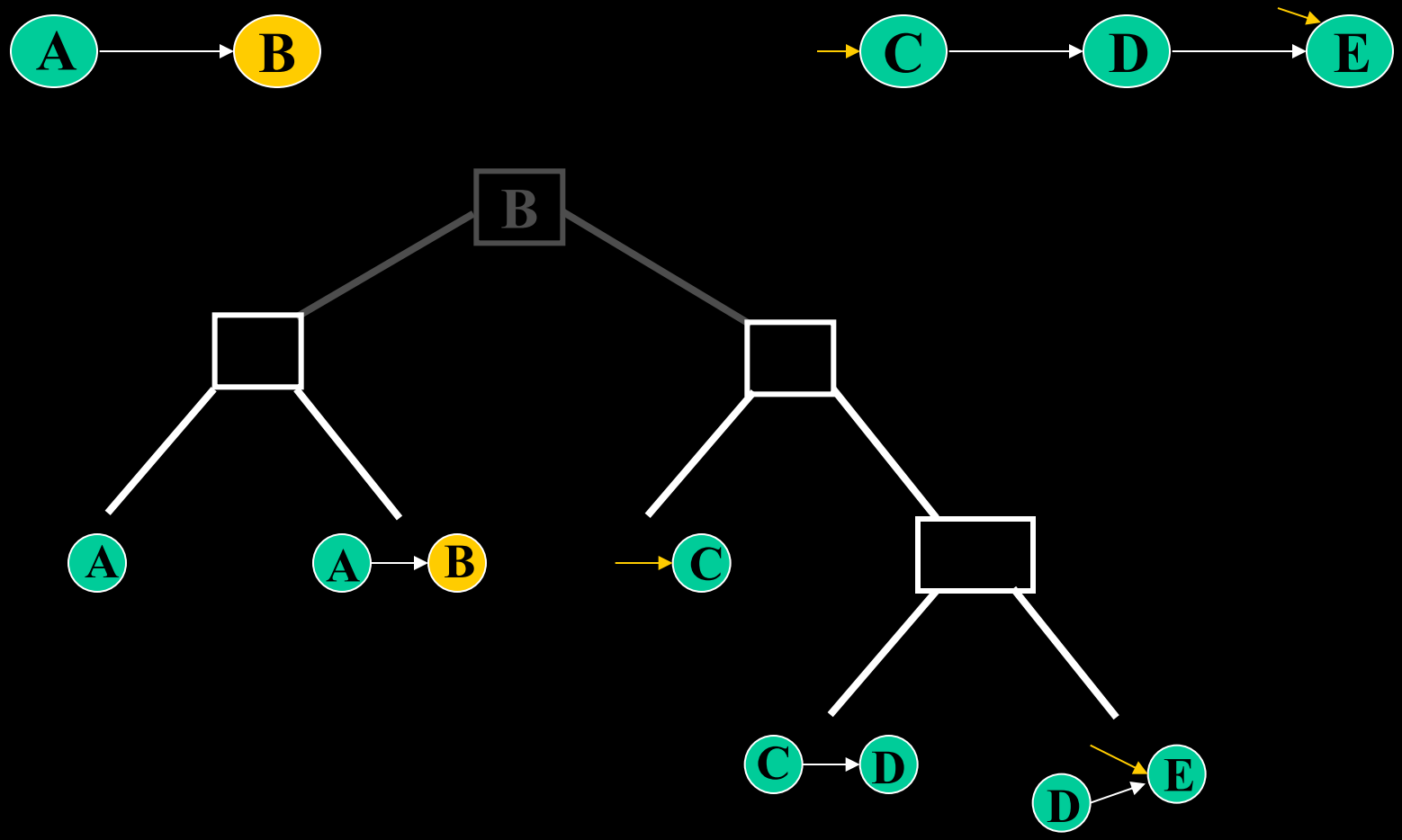
Decomposition Tree



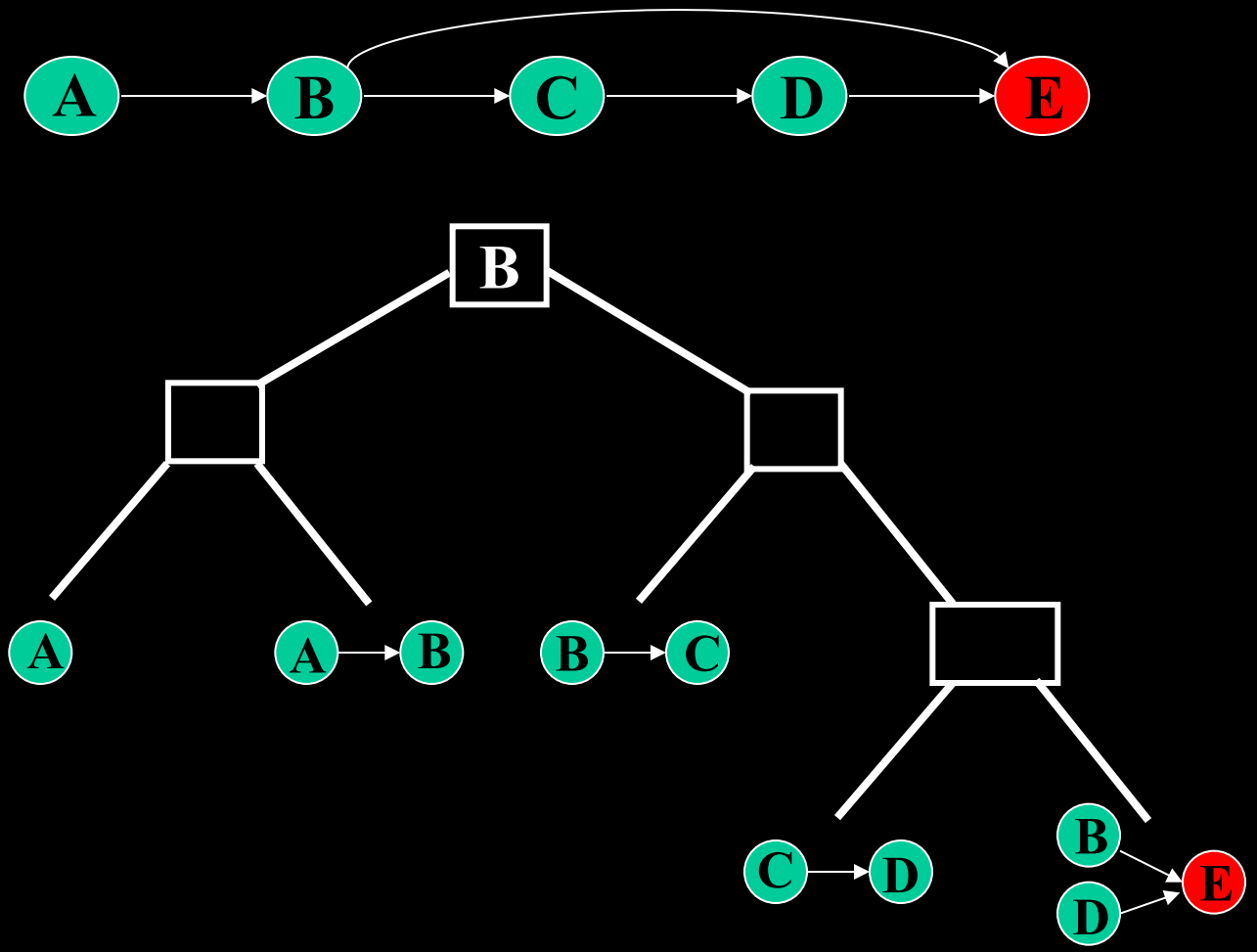
Decomposition Tree



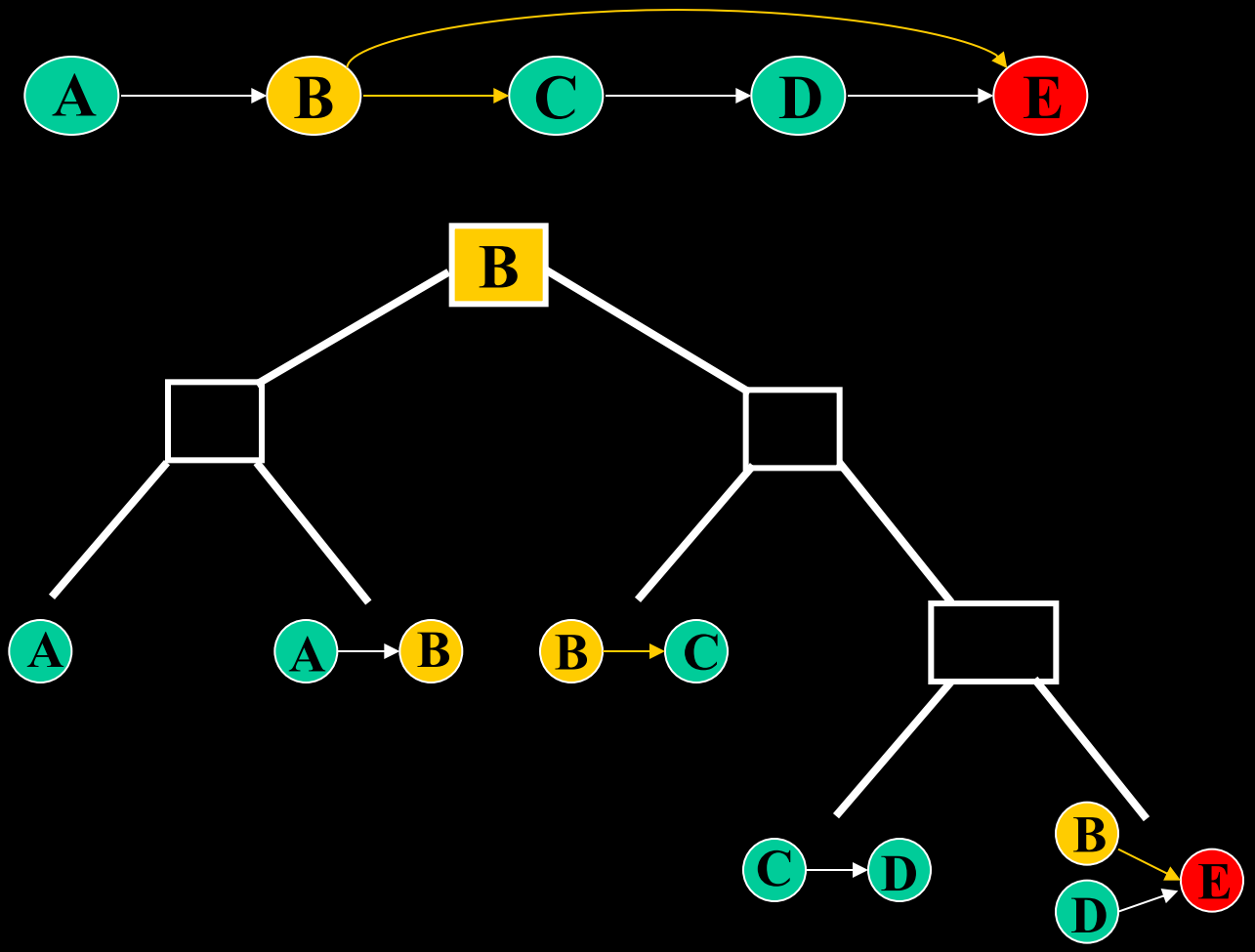
Decomposition Tree



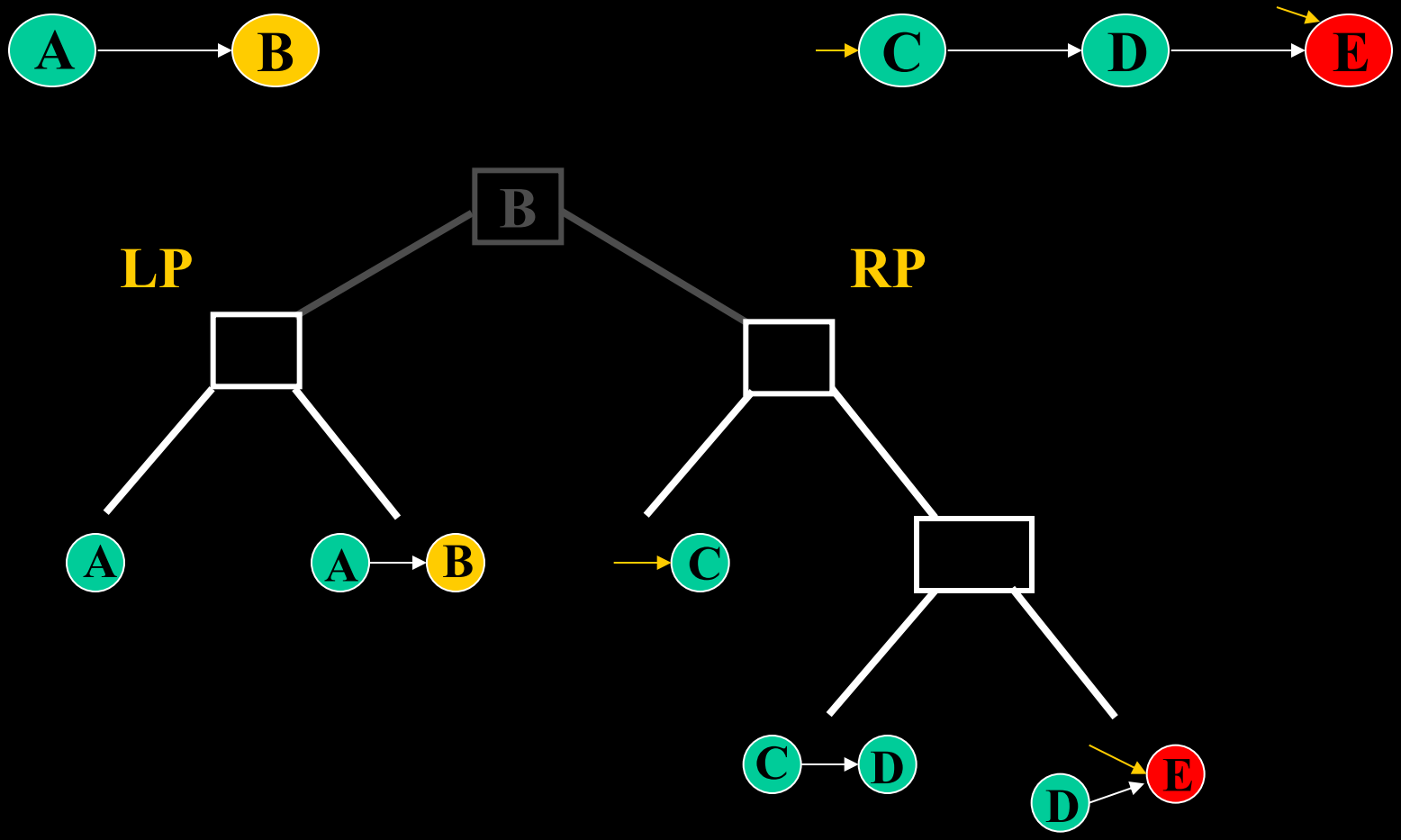
Decomposition Tree



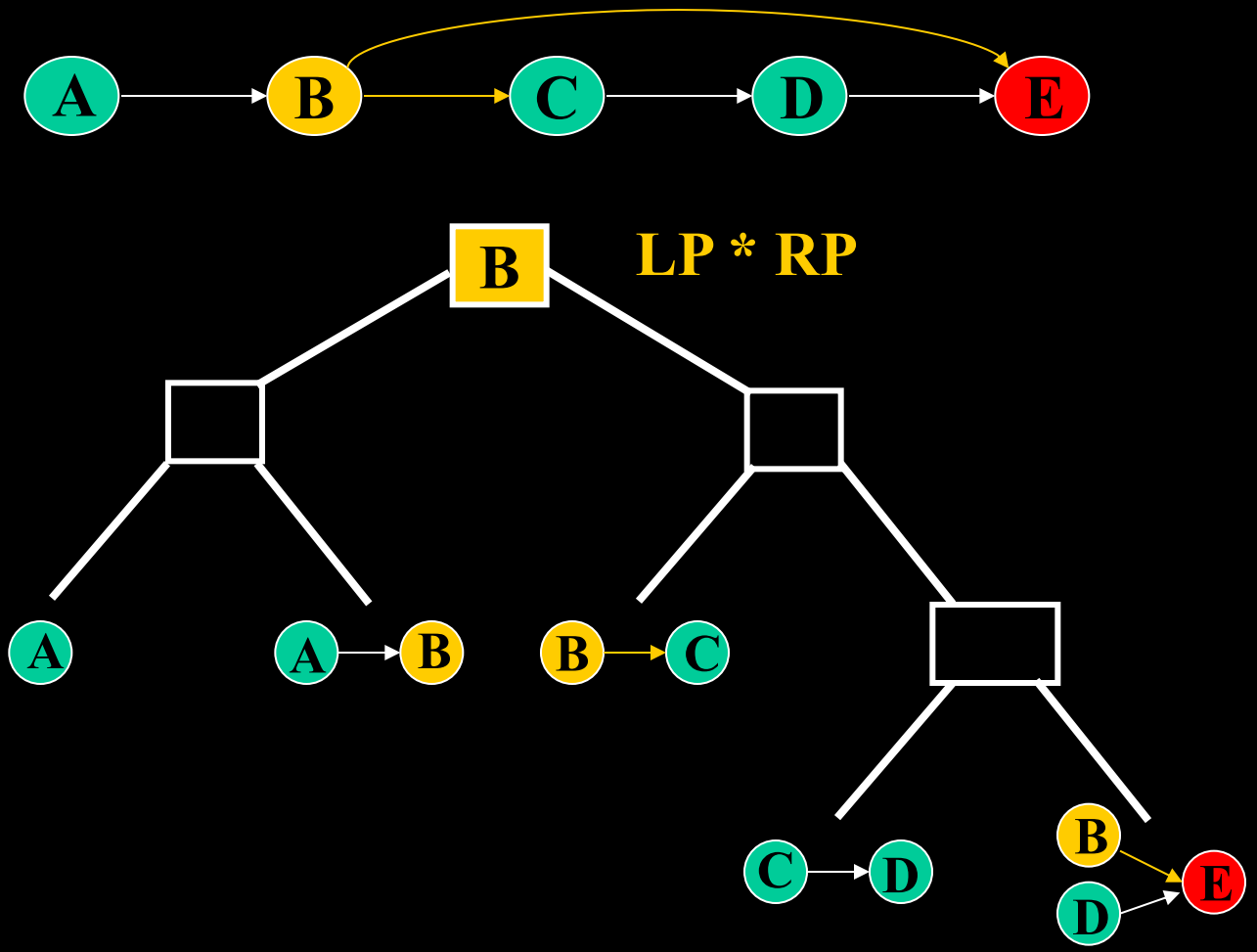
Decomposition Tree



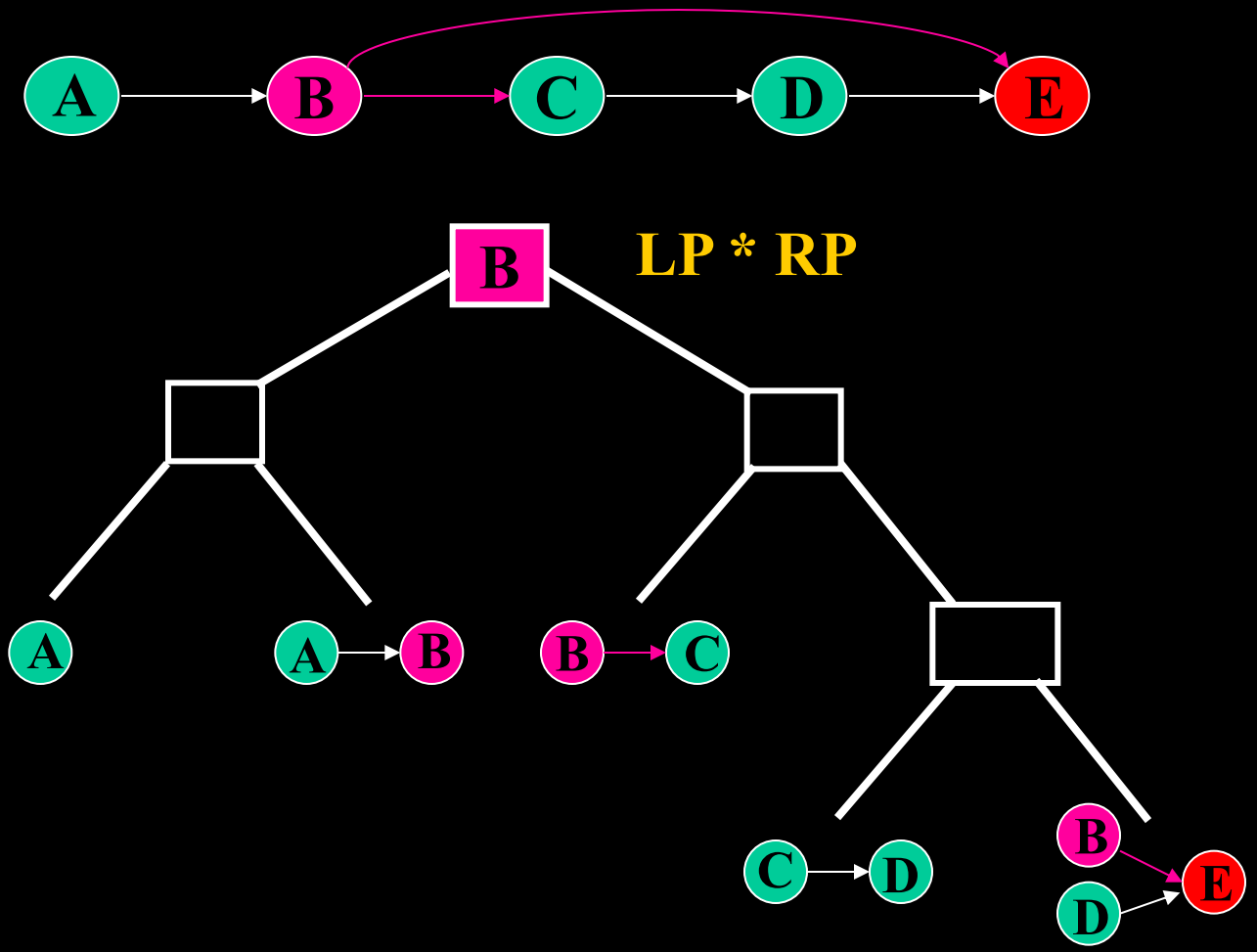
Decomposition Tree



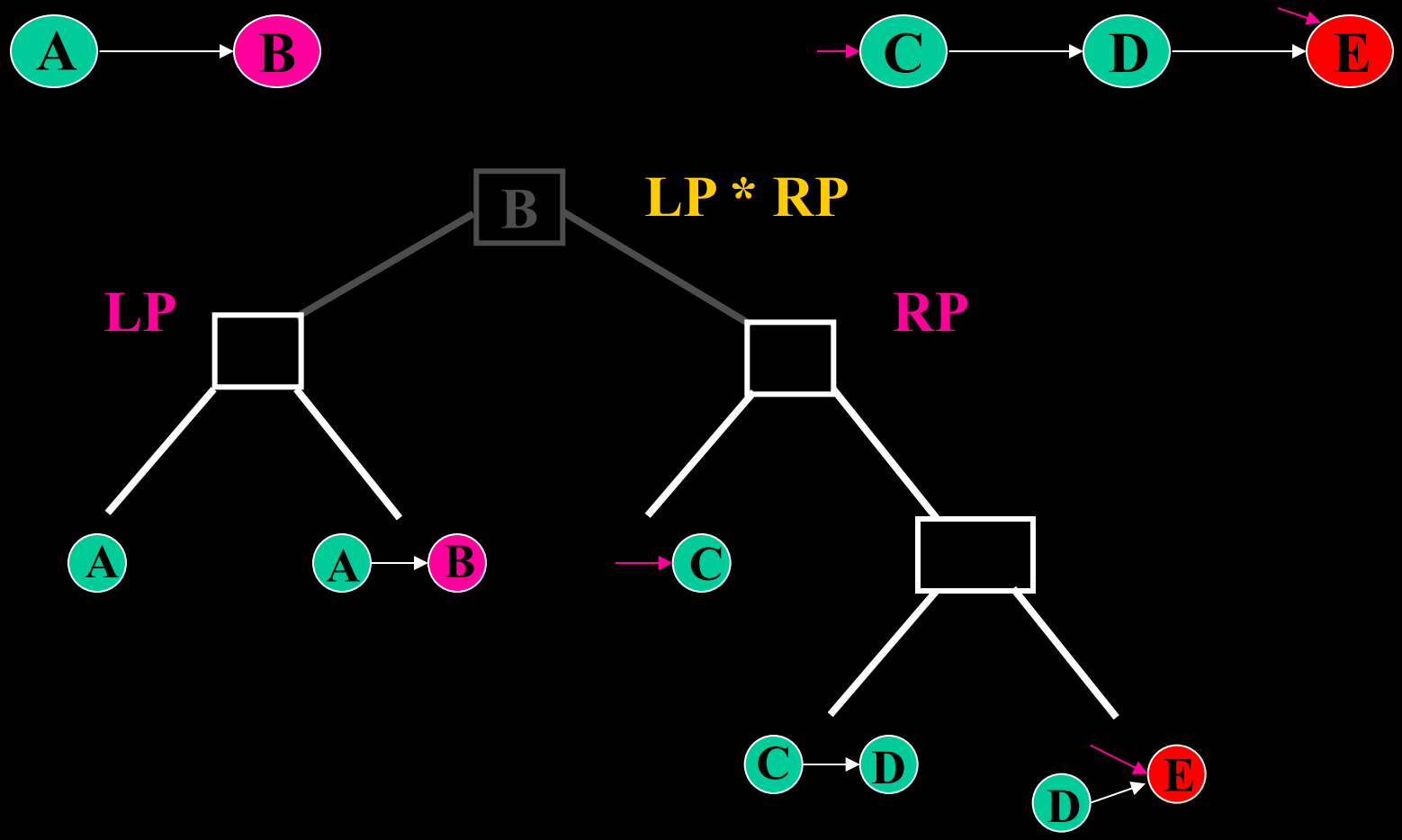
Decomposition Tree



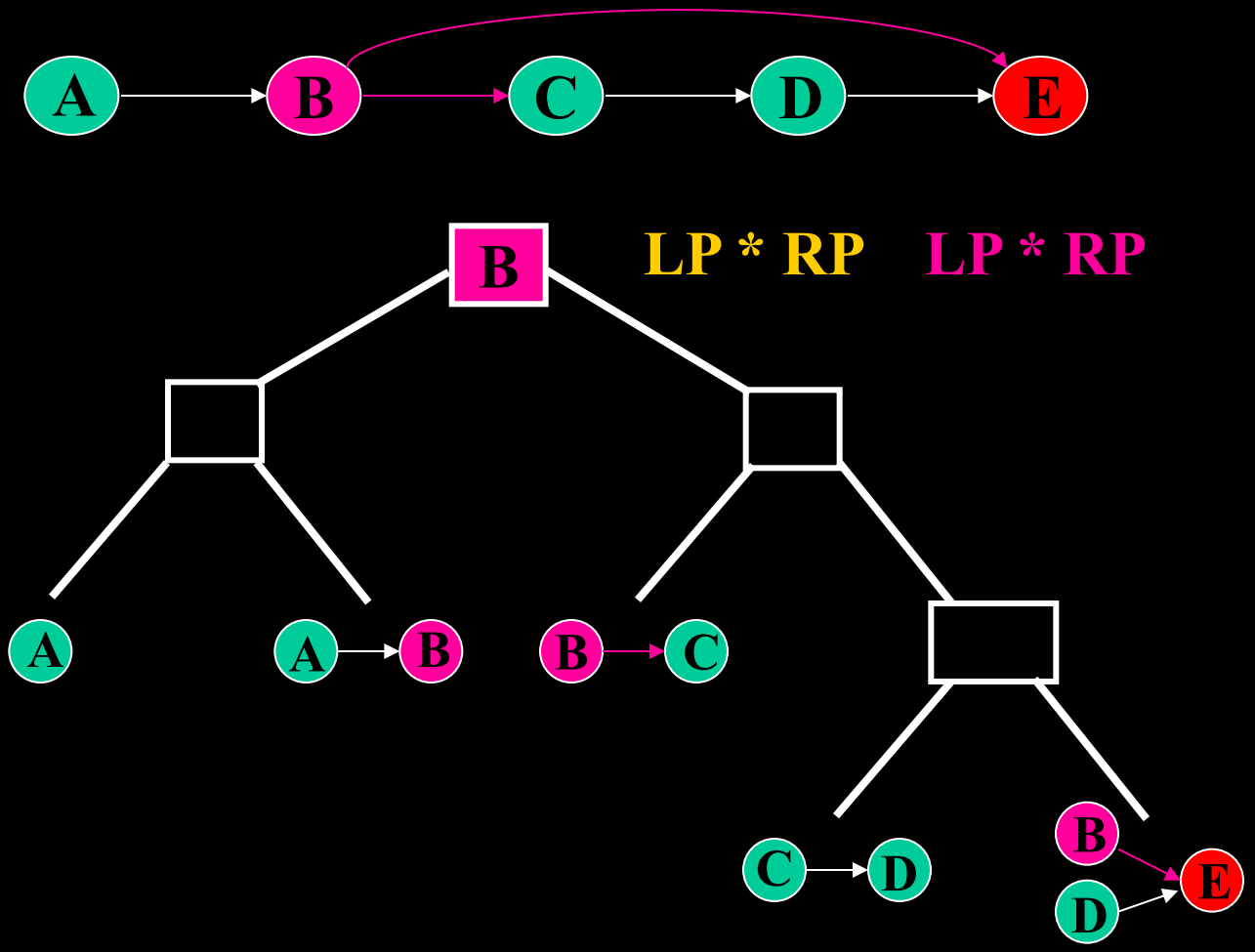
Decomposition Tree



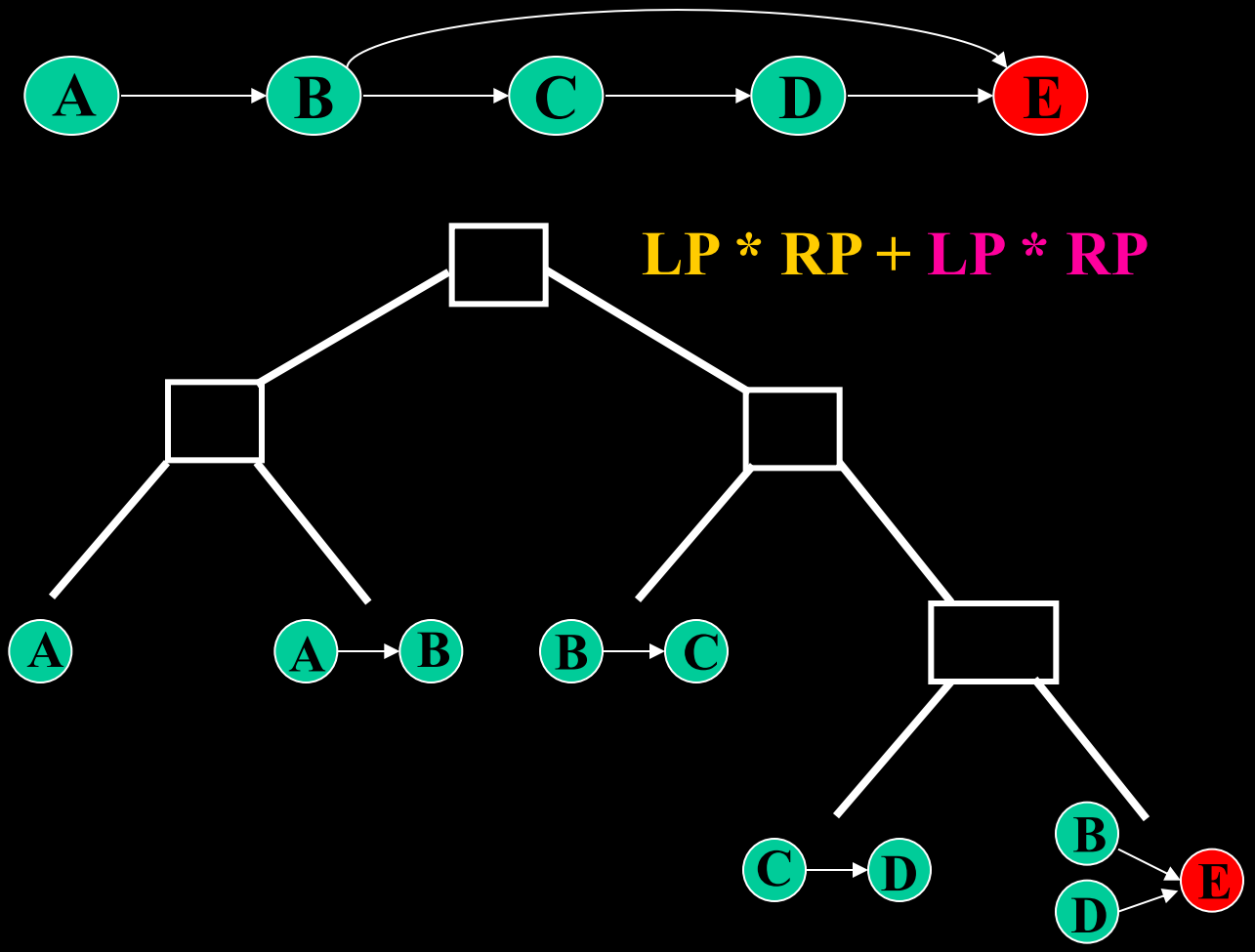
Decomposition Tree



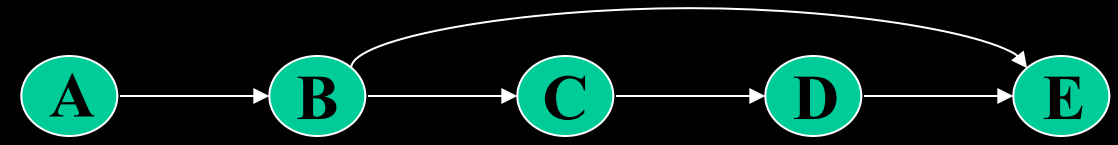
Decomposition Tree

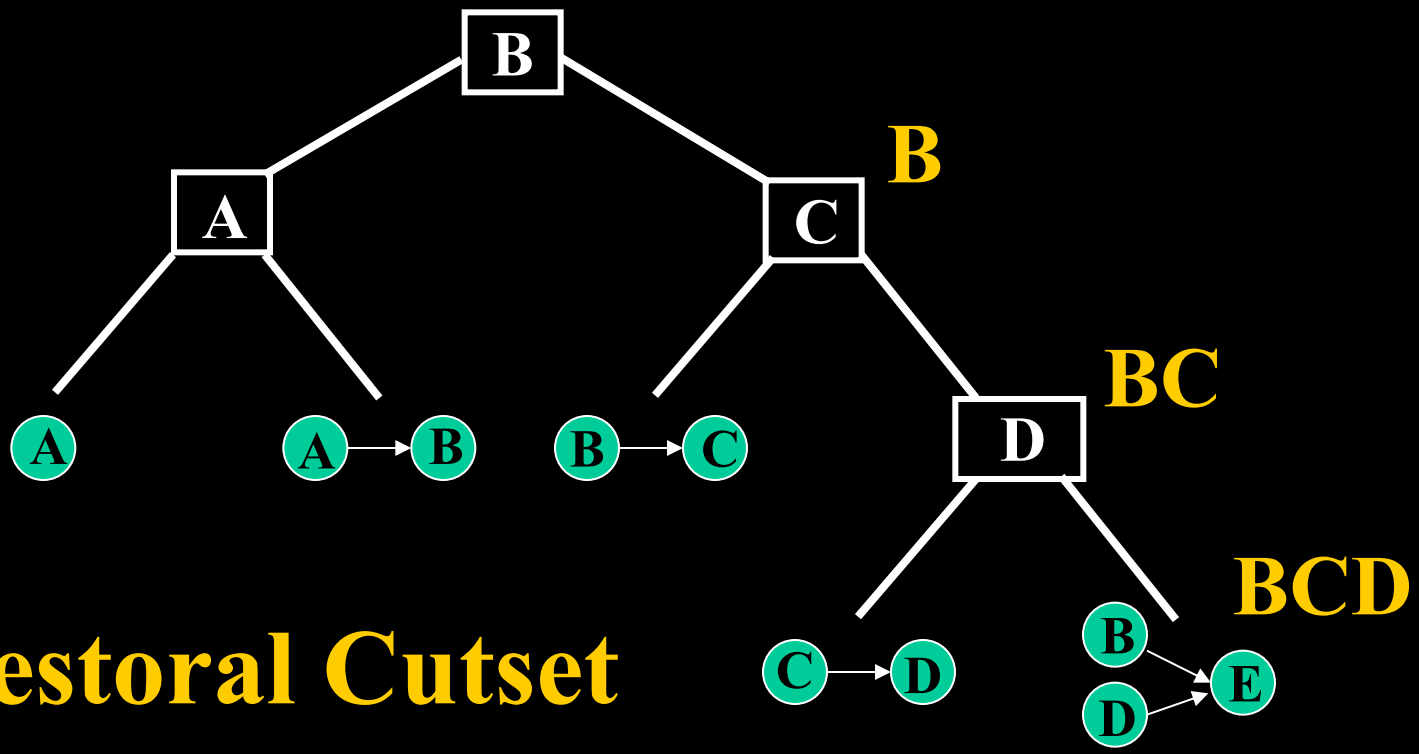


Decomposition Tree



Computational Complexity

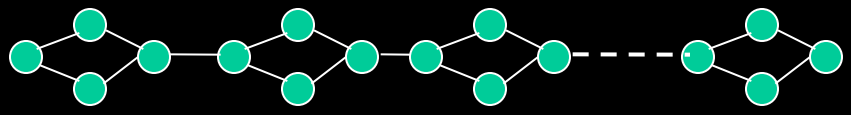




Ancestoral Cutset

Computational Complexity

- Given
  - DAG with **n** nodes
  - elimination order of width **w**

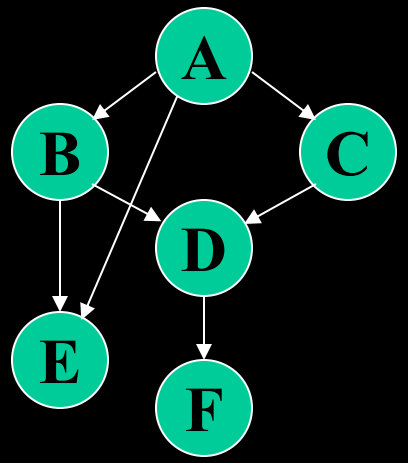


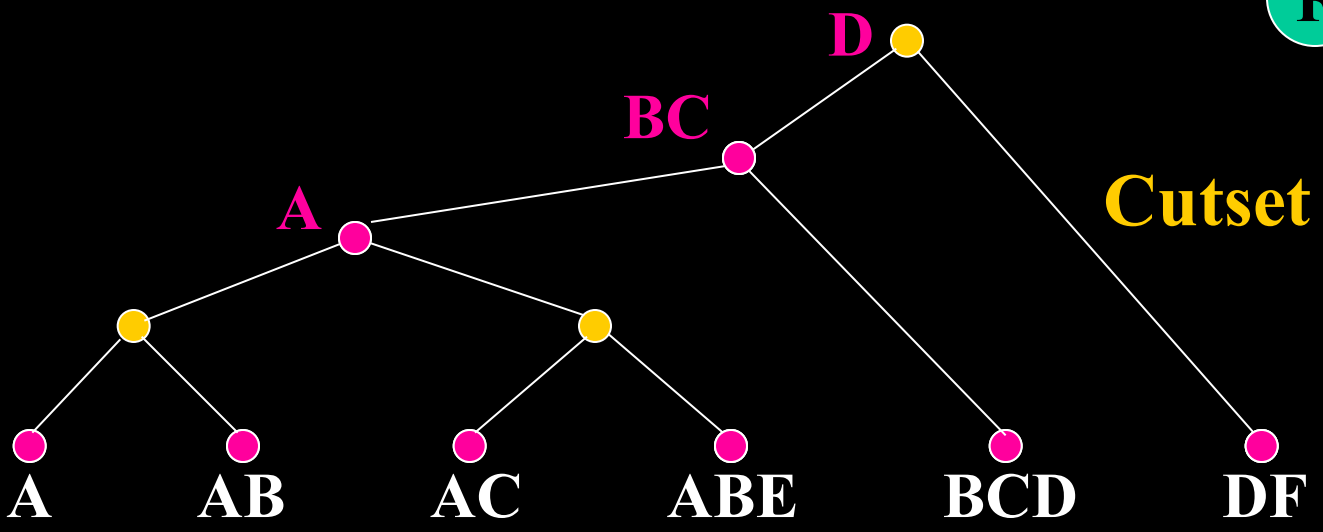
- Can construct a dtree in O(n log n) time:
  - Height **O(log n)**
  - cutset width **<= w+1**
  - a-cutset width **O(w log n)**
- Time complexity: $\mathbf{O(n\ c^{w \log n})}$
- Cutset Conditioning: $\mathbf{O(n\ c^{S})}$

	Network	Effective Network Size	Elimination-Order Width	Loop-Cutset Width	A-Cutset Width
1	Water	59.0	21.3	29.5	32.3
2	Midlew	108.4	19.7	39.3	45.9
3	Barley	138.0	21.8	57.3	51.1
4	Diabetes	1349.1	19.2	557.2	77.9
5	Link	922.7	28.0	347.0	70.7
6	Pigs	699.0	16.4	144.2	38.0
7	Munin1	410.3	26.4	122.6	51.7
8	Munin2	2202.7	20.0	495.9	54.6
9	Munin3	2293.4	16.3	454.2	51.1
10	Munin4	2313.4	19.7	521.5	51.9

Constructing dtrees

F E A B C D W=2

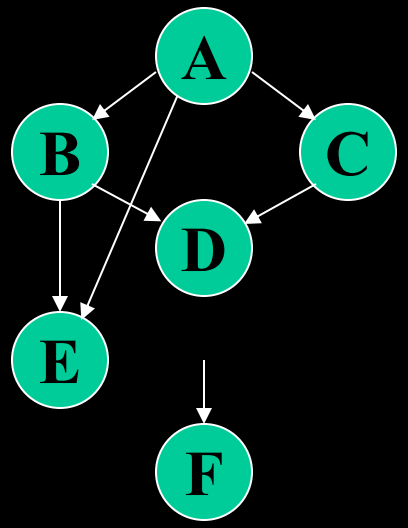


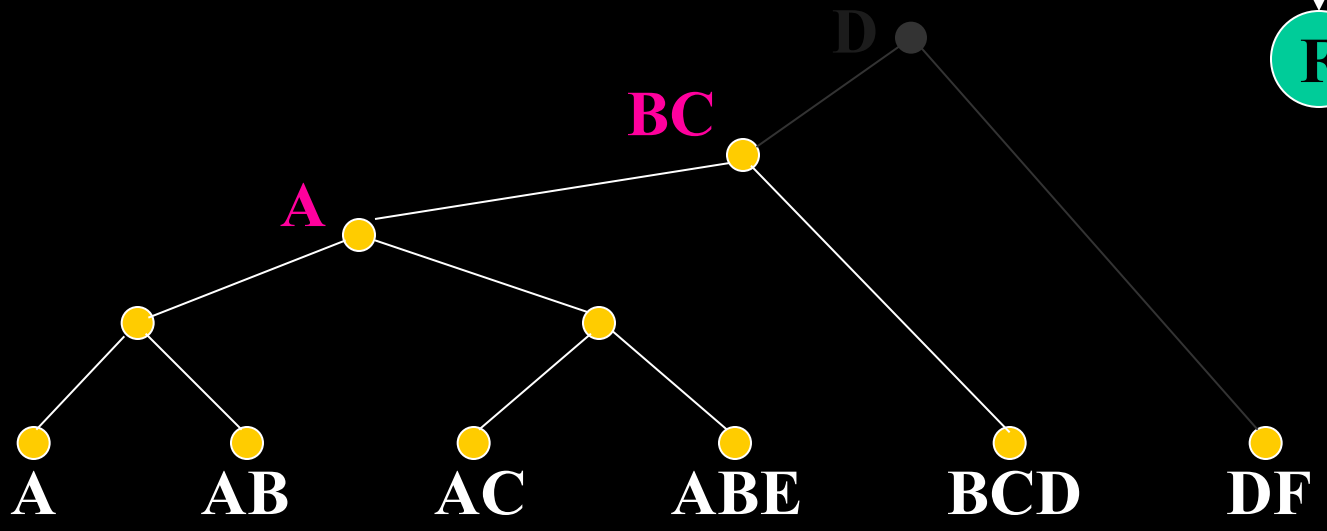


Cutset <= w+1

Constructing dtrees

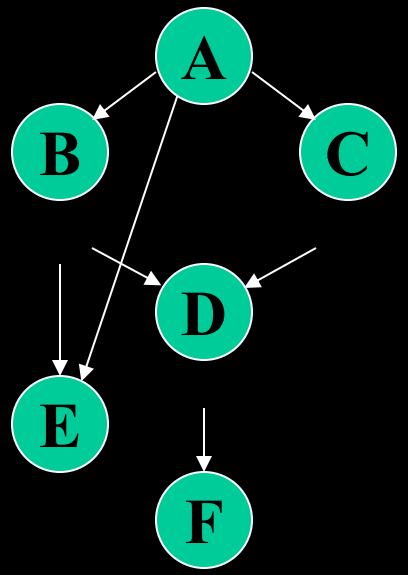
F E A B C D W=2

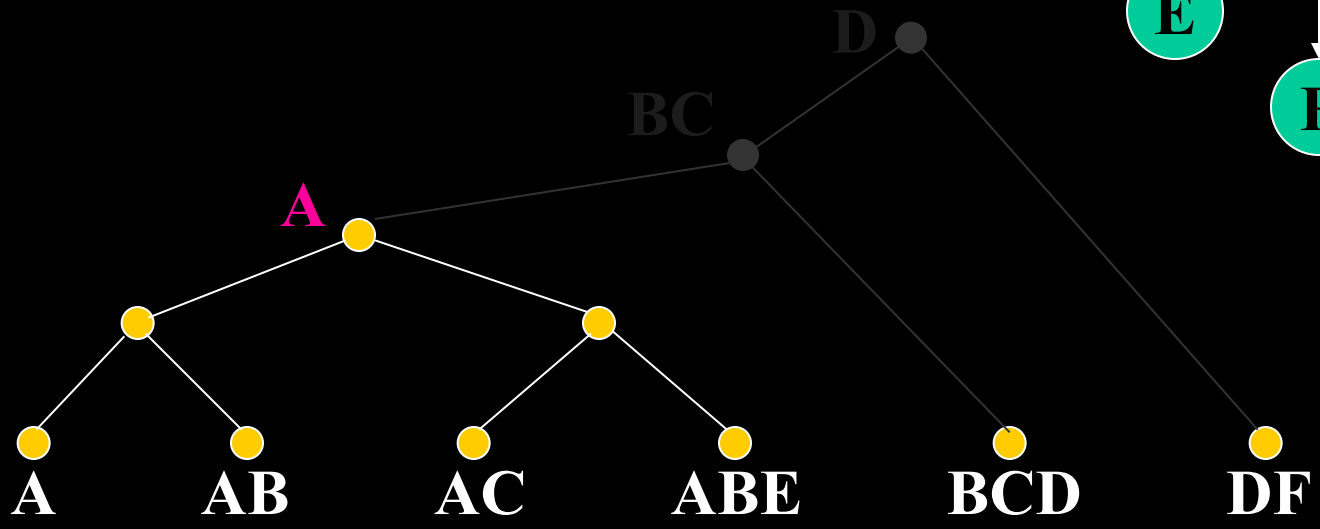




Constructing dtrees

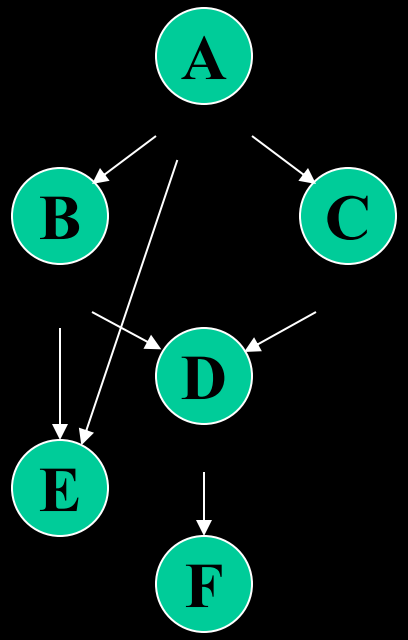
F E A B C D W=2

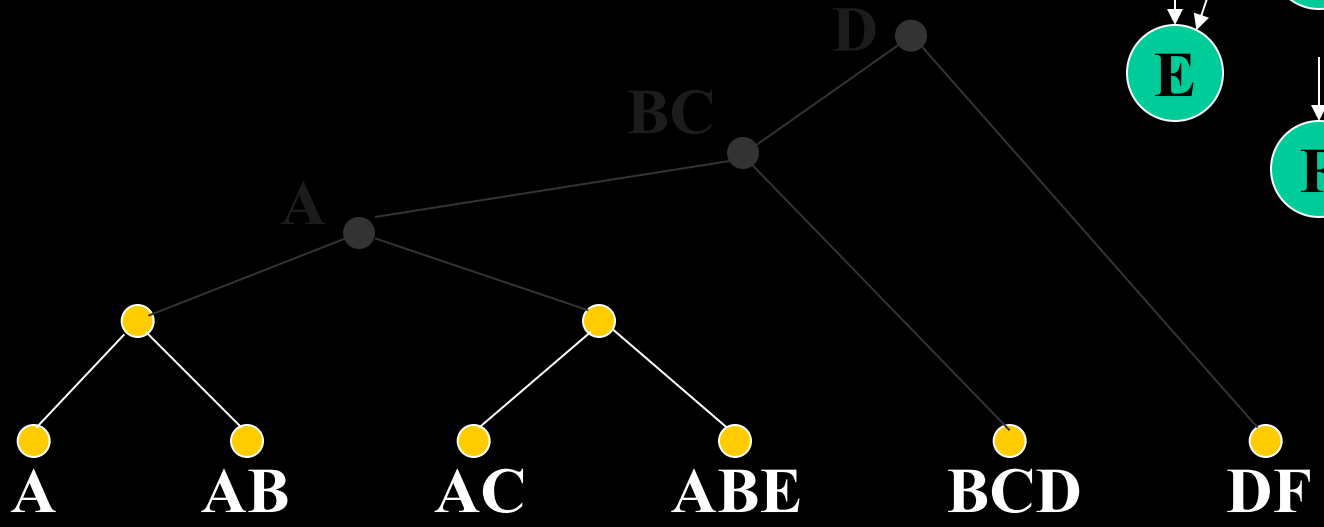




Constructing dtrees

F E A B C D W=2





Relation to Jointrees

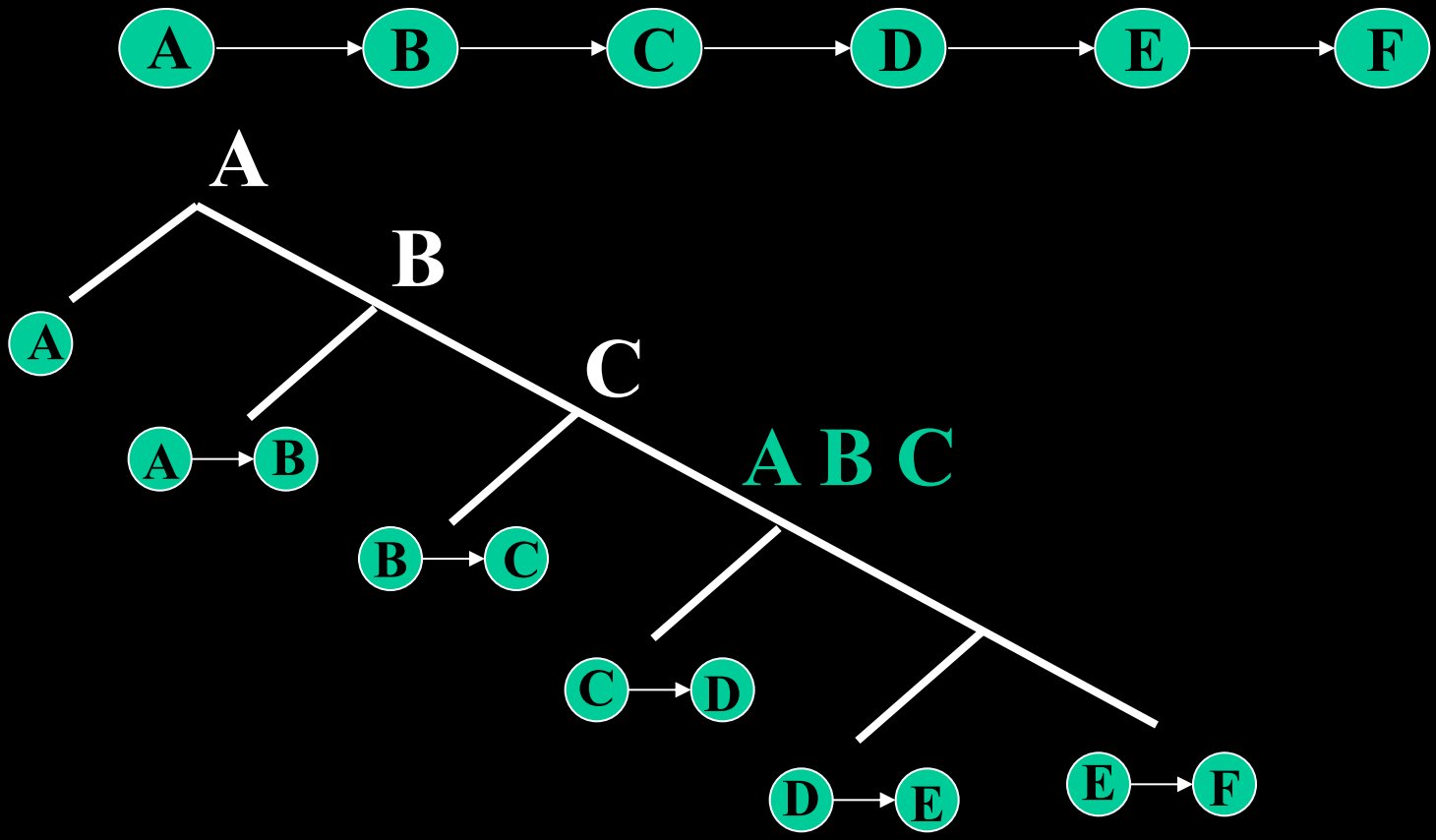
Time: $O(n\, c^{w \log n})$

Space: ~~$O(n)$~~

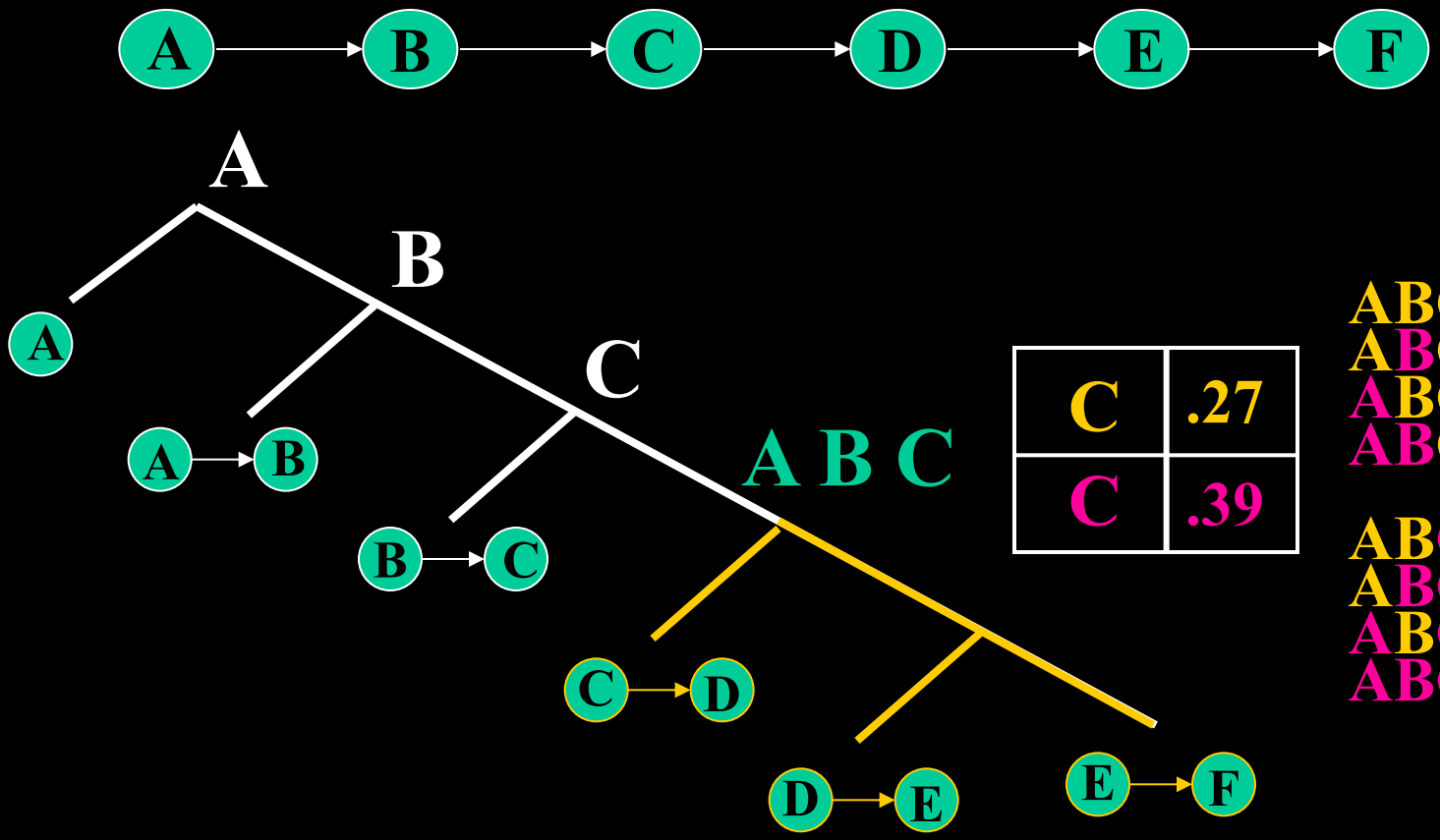
Time: $O(n\, c^{w})$

Space: $O(n\, c^{w})$

Decomposition Tree

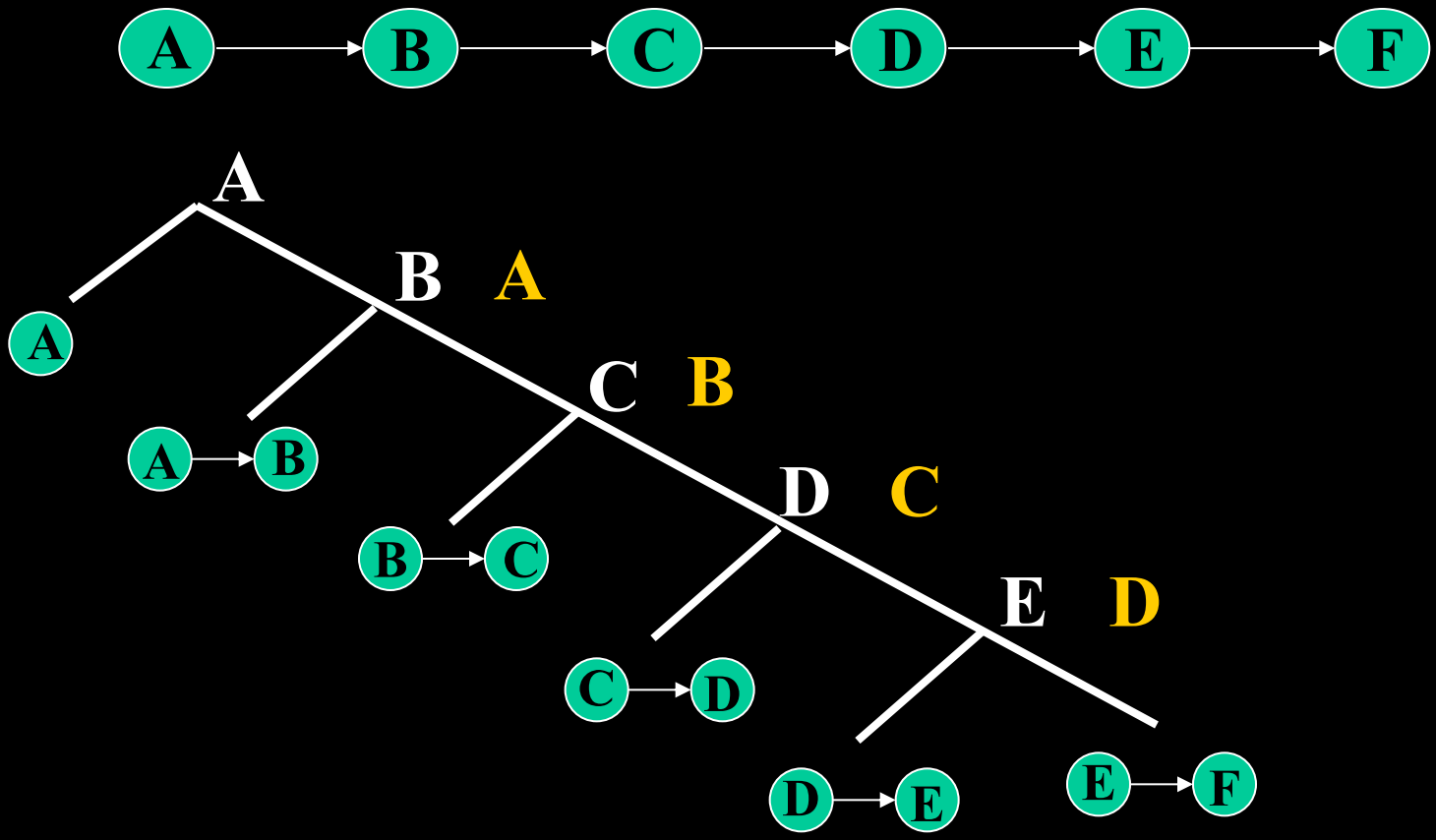


Decomposition Tree



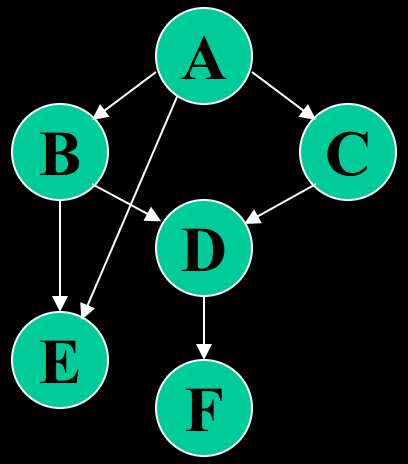
Context(N)= A-Cutset(N)&Vars(N)

Decomposition Tree

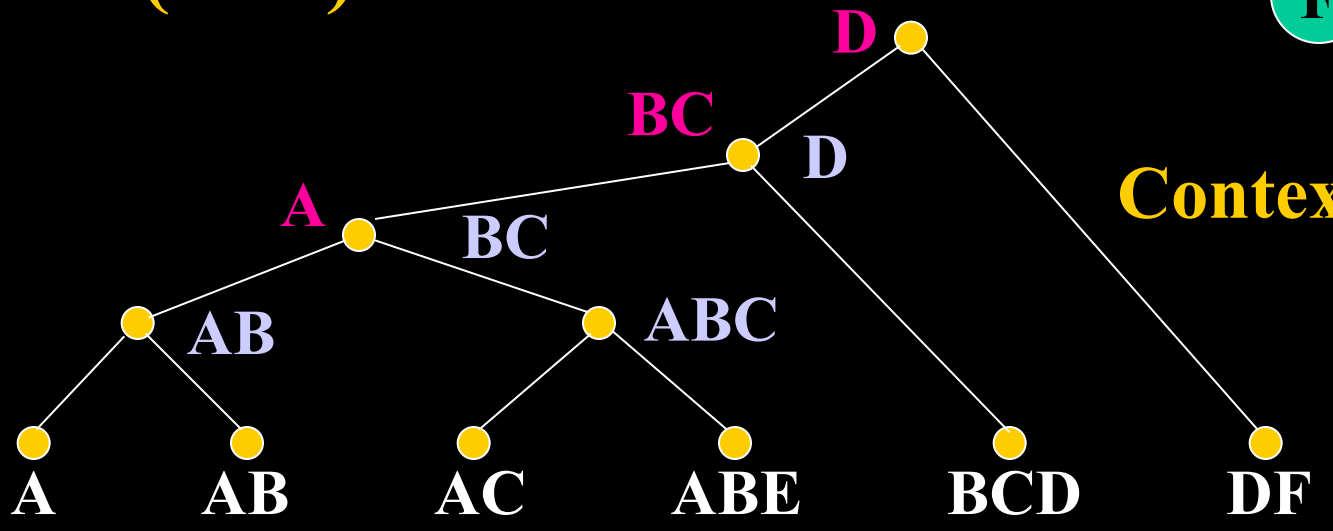


Constructing dtrees

F E A B C D W=2



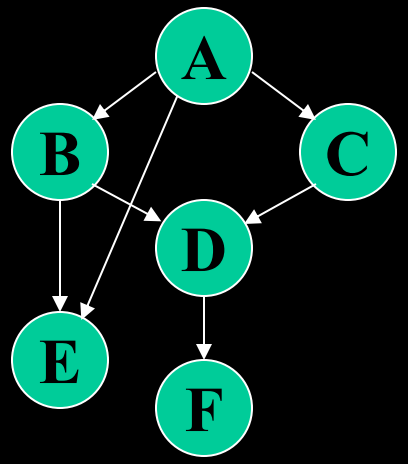
Space: $O(n\ c^{w})$



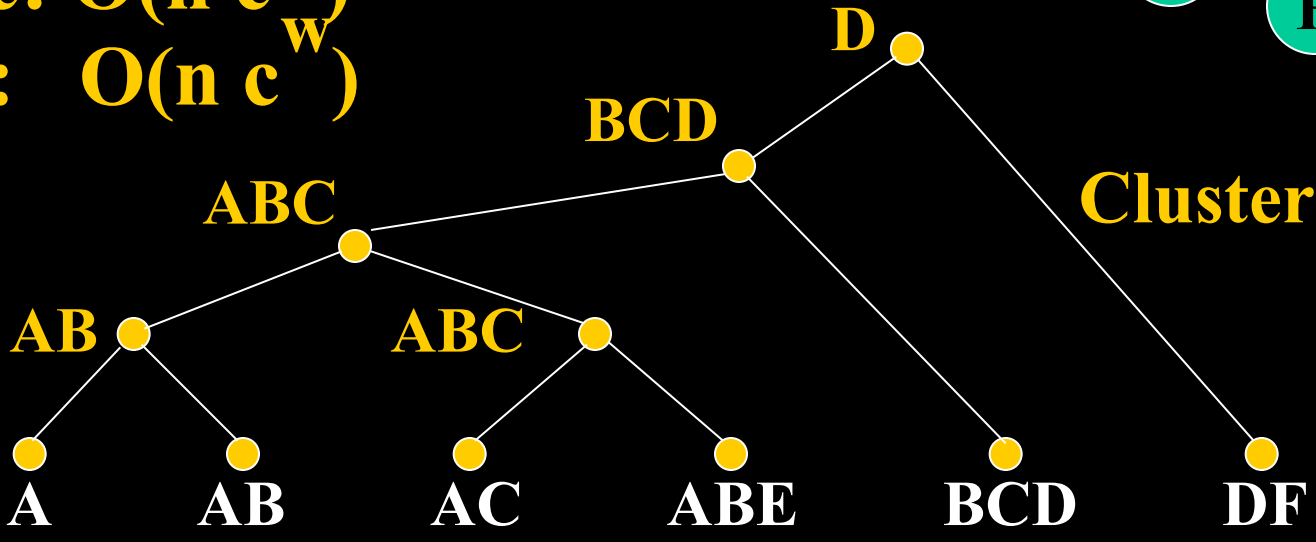
Context <= w+1

Constructing dtrees

F E A B C D W=2



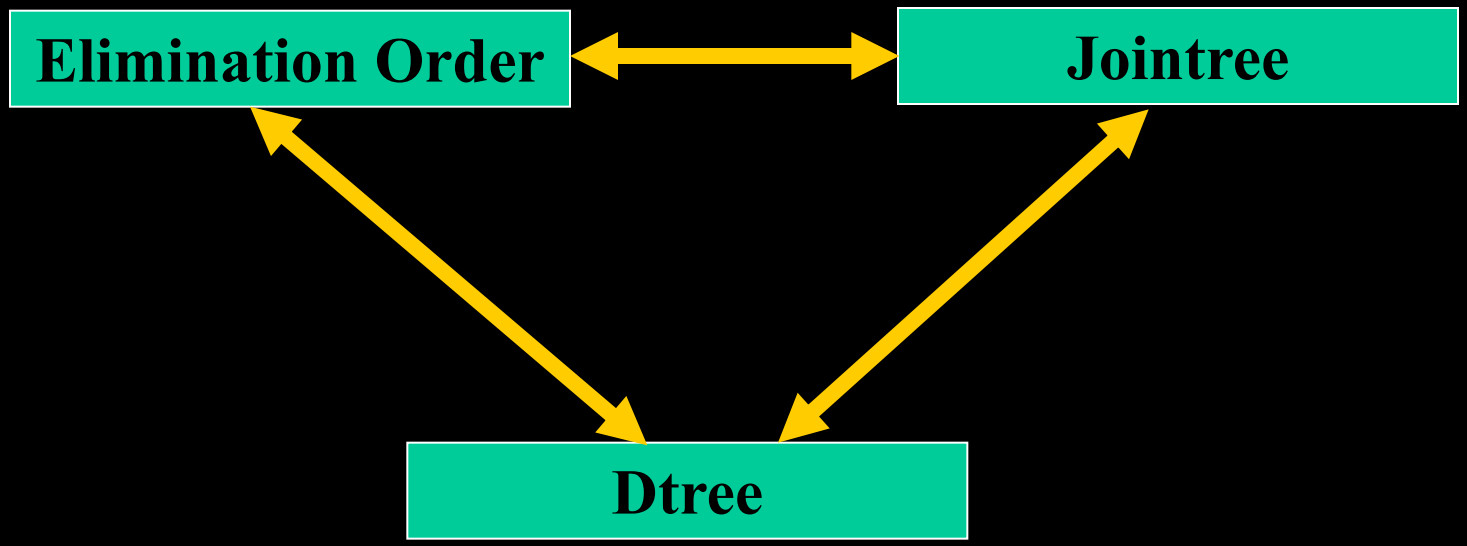
Space: $O(n\ c^{w})$
Time: $O(n\ c^{w})$



Cluster <= w+1

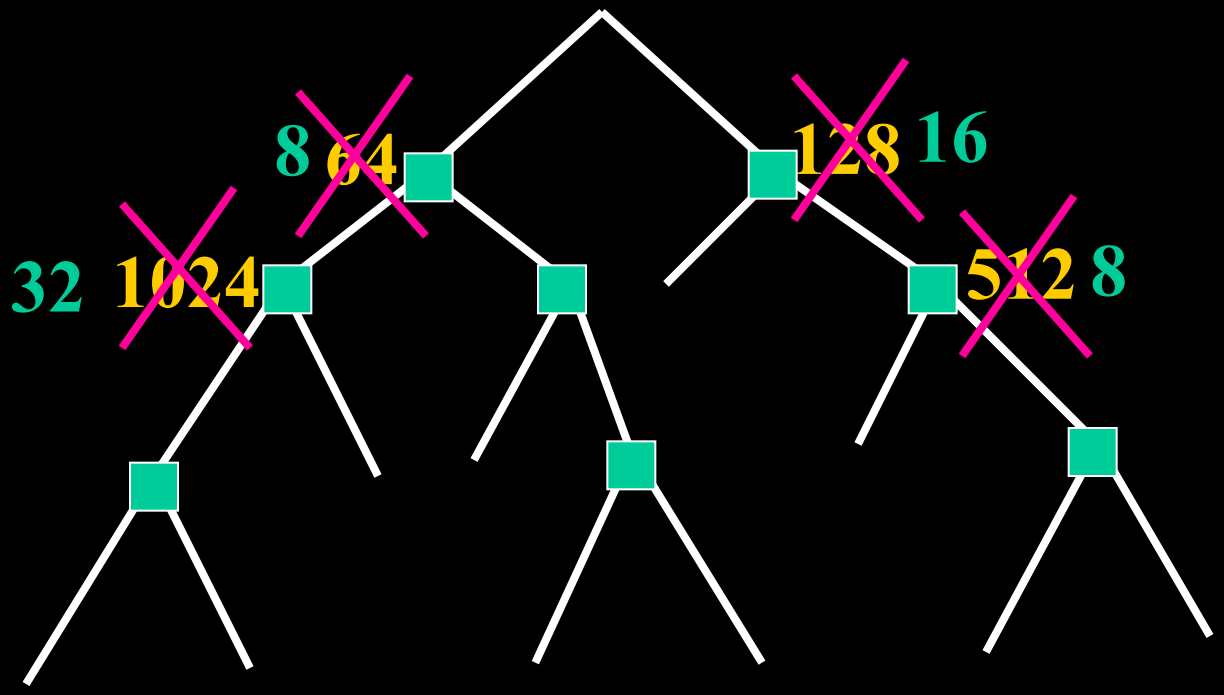
Decomposition Models

width-preserving transformations



Any-Space Inference

Time-Space Tradeoffs



1728 cache entries

64 cache entries

$$rc(T)=cutset^{\#}(T^p)[cf(T^p)context^{\#}(T^p)+(1-cf(T^p))rc(T^p)]$$

Conclusion

- Alternative conditioning paradigm
  - condition to decompose
  - dtrees: decomposition policy

- Any-space inference:

  $O(n)$ ---- $O(n\ c^{w})$

  $O(n\ c^{w \log n})$ ---- $O(n\ c^{w})$

Conclusion

- Three decomposition models:
  - elimination order (variable elimination)
  - jointree (clustering)
  - dtree (conditioning)
- Quality measured by width
- Poly-time, width-preserving transformations