

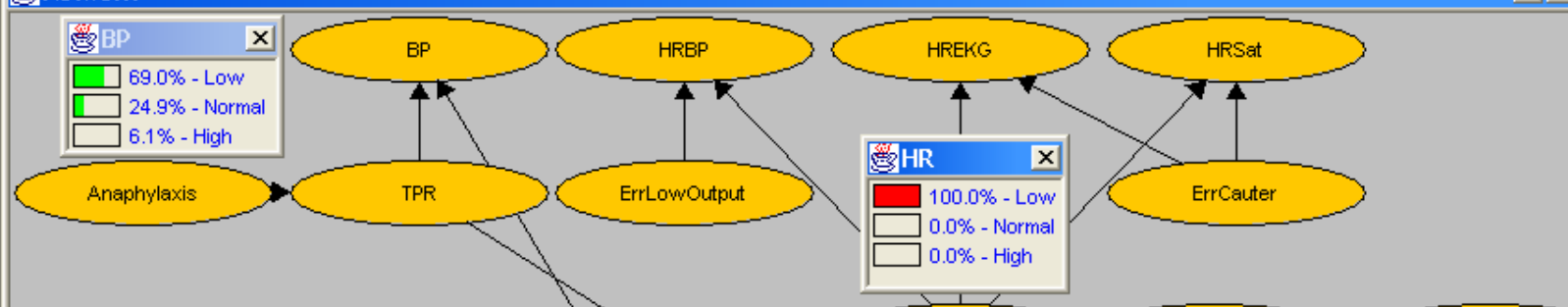
Sensitivity Analysis in Bayesian Networks

Chapter 16

AUXILIARY NODES

- + Anaphylaxis
- + ArtCO2
- + BP
- + Catechol
- + CO
- + CVP
- + Disconnect
- + ErrCauter
- + ErrLowOutput
- + ExpCO2
- + FiO2
- + History
- + **HR = Low**
- + HRBP
- + HREKG
- + HRSat
- + Hypovolemia
- + InsuffAnesth
- + Intubation
- + KinkedTube
- + LVEDVolume
- + LVFailure
- + MinVol
- + MinVolSet
- + PAP
- + PCWP
- + Press
- + PulmEmbolus
- + PVSat
- + SaO2
- + Shunt
- + StrokeVolume
- + TPR
- + VentAlv
- + VentLung
- + VentMach
- + VentTube

Network



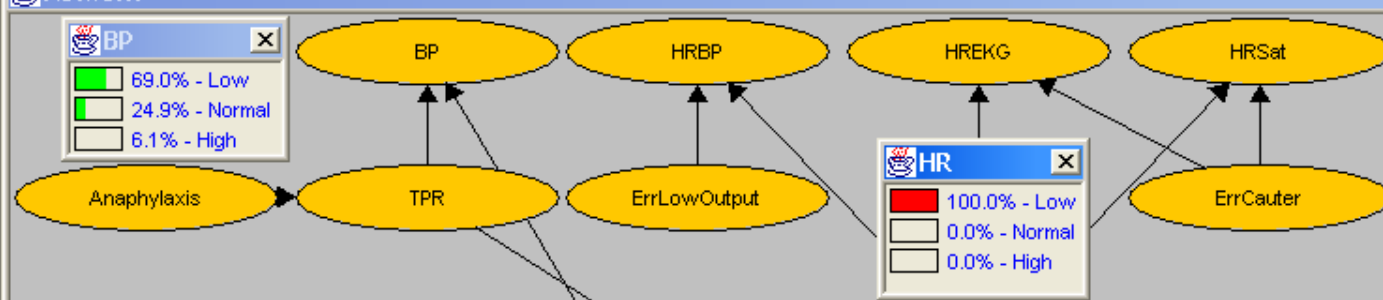
Now $\Pr(\text{BP} = \text{Low} \mid \text{HR} = \text{Low}) = 0.69$,
but experts believe it should be ≥ 0.75 .

How do we efficiently identify minimal
parameter changes that can help us
satisfy this query constraint?

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Network



Sensitivity analysis

Event

BP

= Pr(Low)=0.689811

Constraint

>= 0.75

Start sensitivity analysis

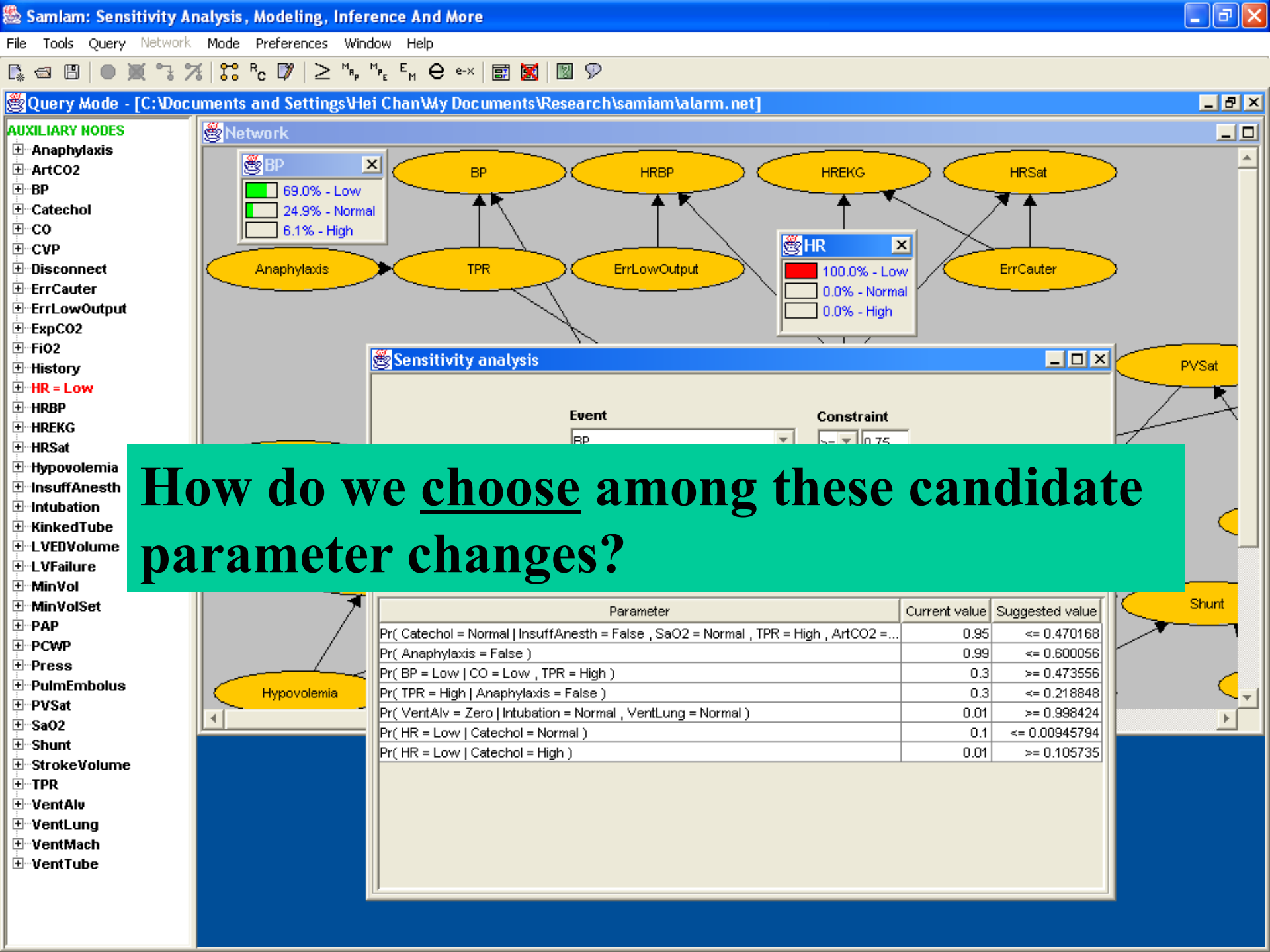
☐ Constrain Two Events

☐ Show Table Details

Edit

Adopt Change

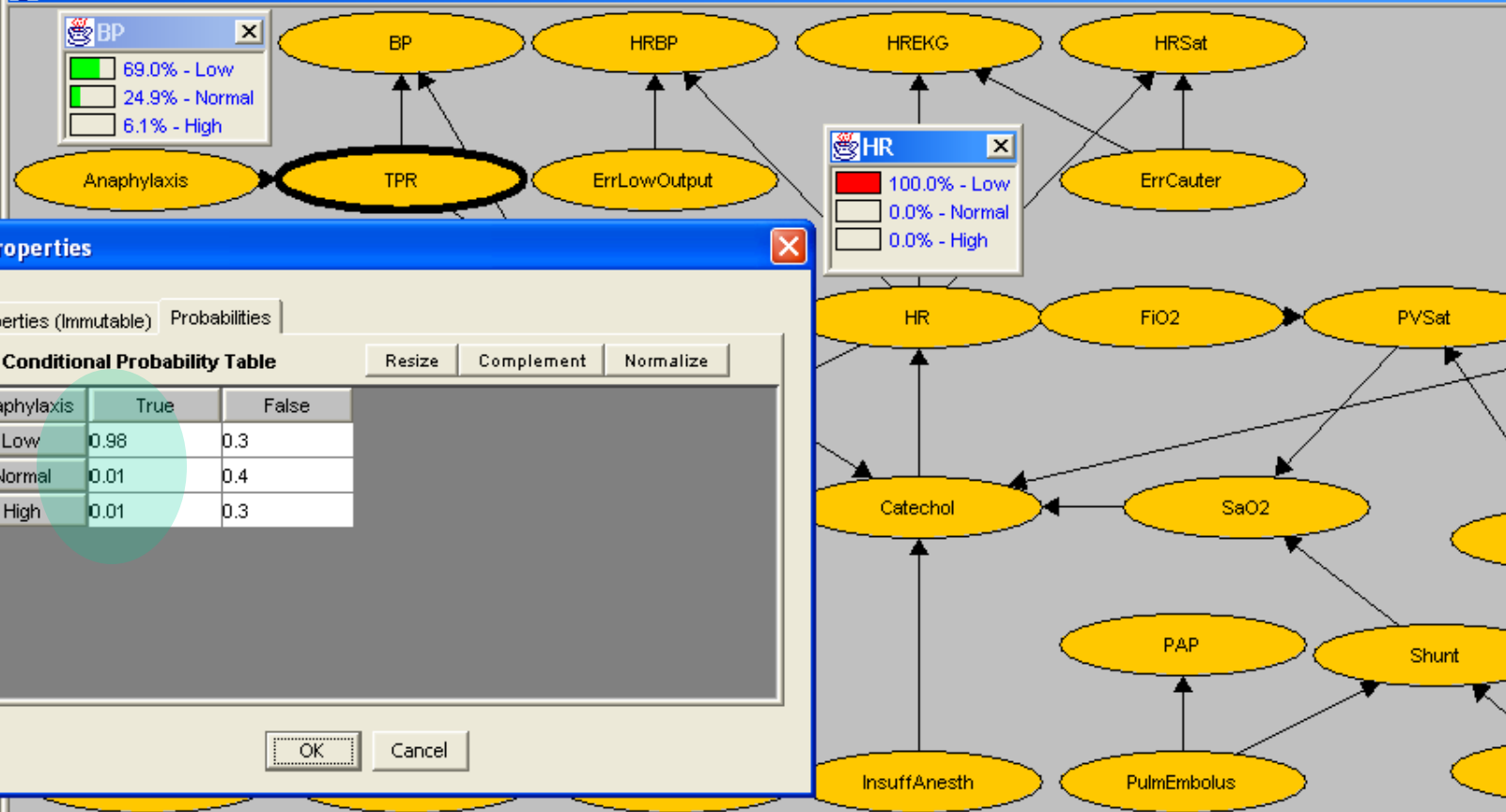
Suggestions given by sensitivity analysis (sort by clicking on a column header)



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TPR Properties

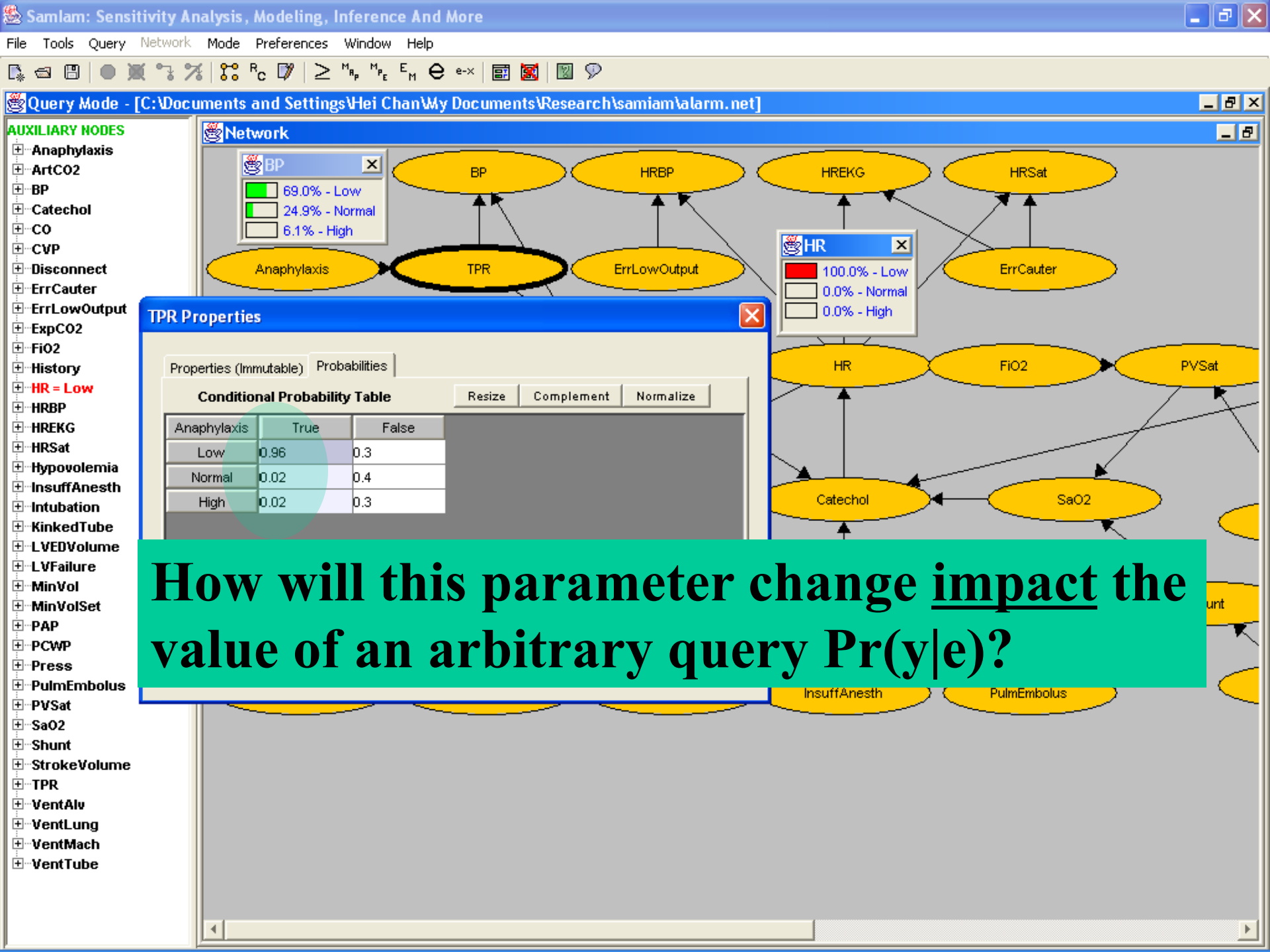
Properties (Immutable) Probabilities

Conditional Probability Table

Anaphylaxis	True	False
Low	0.98	0.3
Normal	0.01	0.4
High	0.01	0.3

Resize Complement Normalize

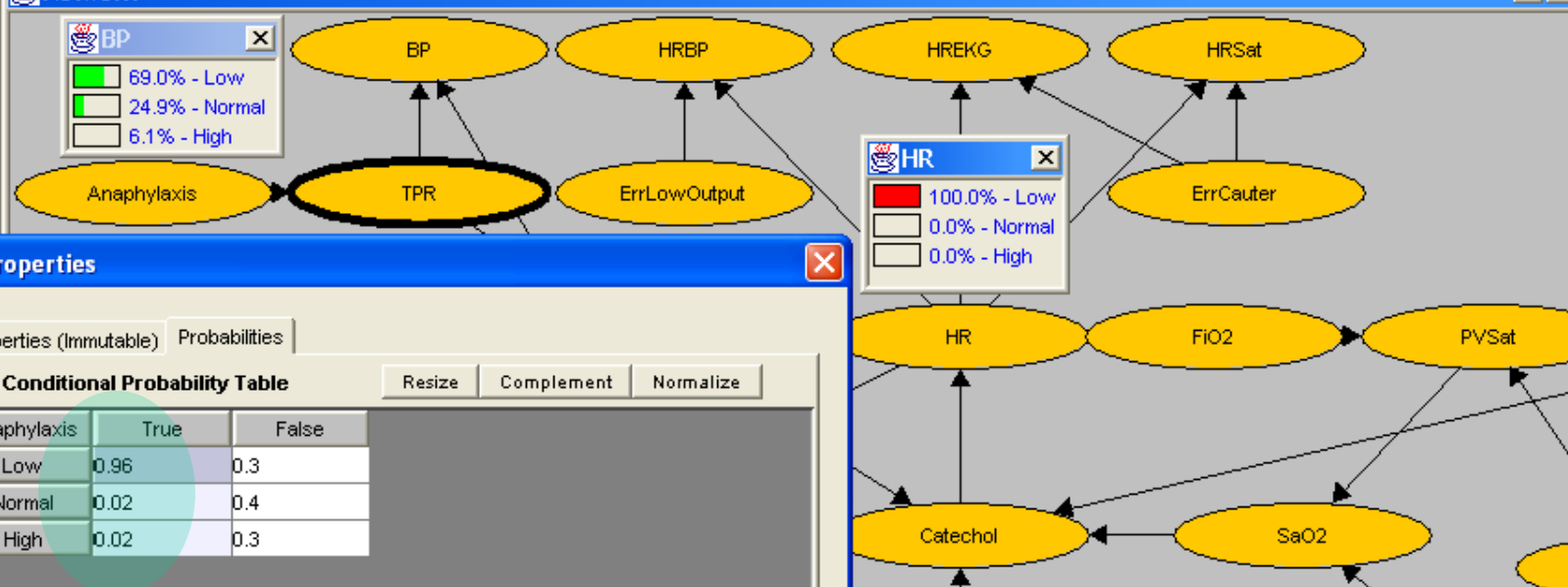
OK Cancel



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How will this parameter change impact the value of an arbitrary query $\Pr(y|e)$?

Questions Addressed

- From global to local belief change
 - How do we efficiently identify parameter changes that can help us satisfy a query constraint?
- Quantifying belief change
 - How do we choose the “smallest” change among several candidate parameter changes?

Applications of Sensitivity Analysis

- Help experts build and debug Bayesian networks, by identifying parameters that must be accurately estimated.
- Estimate new query results after parameter changes without redoing inference.
- Approximate a Bayesian network with a simpler one, and provide guarantees on the query results.

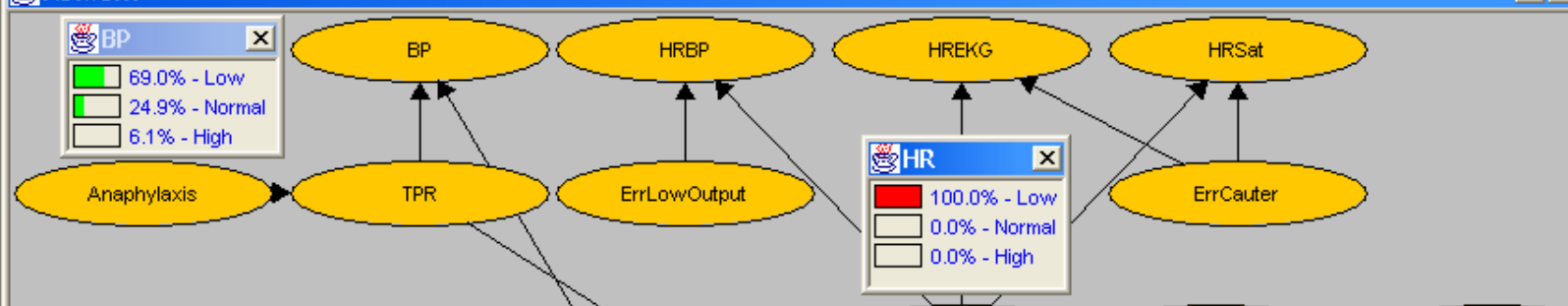
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How do we efficiently identify minimal
parameter changes that can help us
satisfy this query constraint?

From Global to Local Belief Change

Goal: for each network parameter $\theta_{x|u}$,
compute the minimal amount of change
that can enforce query constraints such as:

1. $\Pr(y|e) \geq \varepsilon$.

2. $\Pr(y|e) - \Pr(z|e) \geq \varepsilon$.

3. $\Pr(y|e) / \Pr(z|e) \geq \varepsilon$.

Computing the Partial Derivative

- For each network parameter $\theta_{x|u}$, we introduce a meta parameter $\tau_{x|u}$, such that:

$$\begin{aligned}\theta_{x|u} &= \tau_{x|u} \\ \theta_{\bar{x}|u} &= 1 - \tau_{x|u}\end{aligned}$$

- Our procedure requires us to first compute the partial derivative $\partial \Pr(e) / \partial \tau_{x|u}$:

$$\alpha_e = \frac{\partial \Pr(e)}{\partial \tau_{x|u}} = \frac{\Pr(e, x, u)}{\theta_{x|u}} - \frac{\Pr(e, \bar{x}, u)}{\theta_{\bar{x}|u}}$$

Parameter Changes

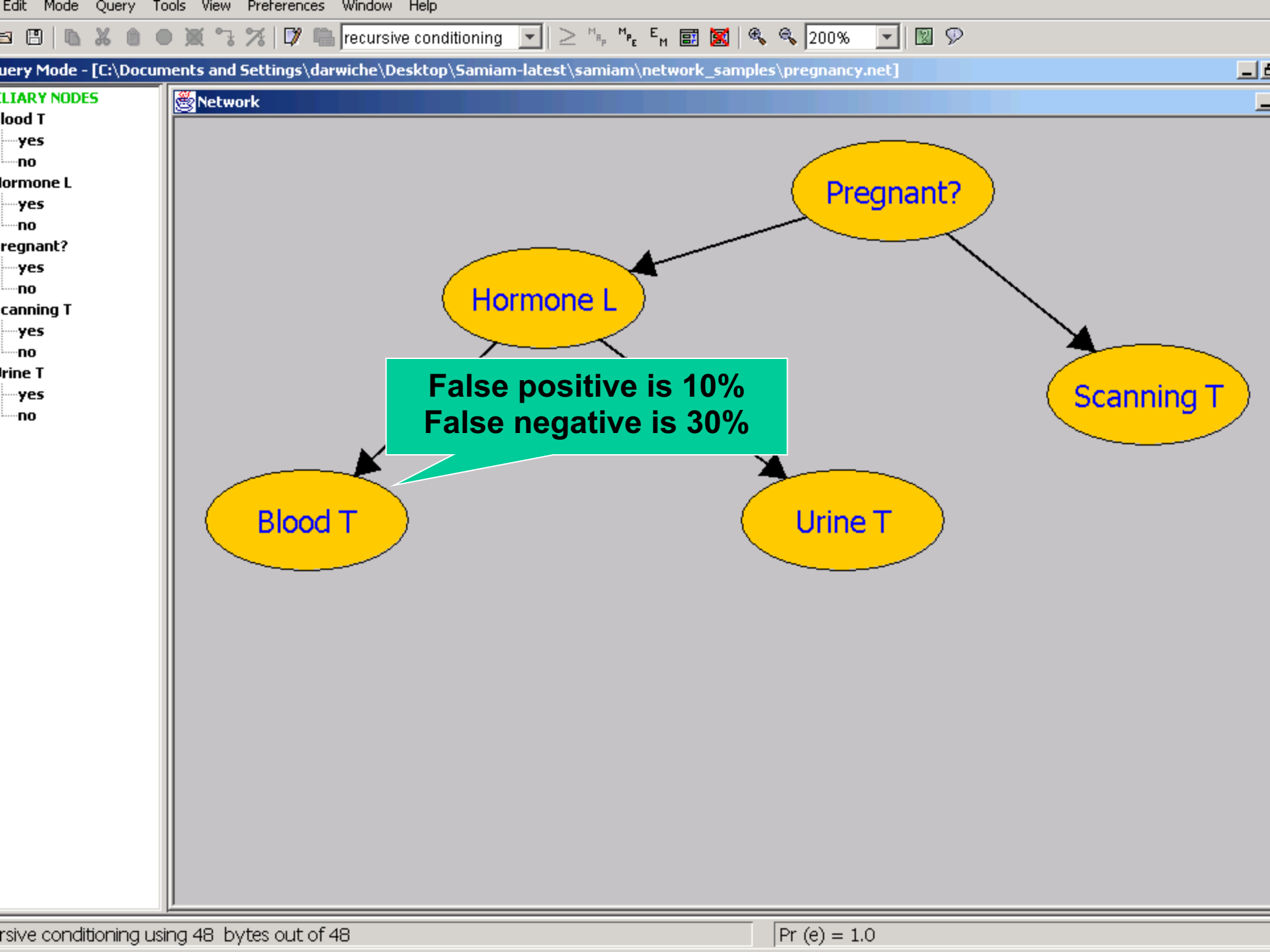
- **Result**: to ensure constraint $\Pr'(y|e) \geq \varepsilon$, we need to change the meta parameter $\tau_{x|u}$ by δ such that:

$$\Pr(y, e) - \varepsilon \Pr(e) \geq \delta [-\alpha_{y,e} + \varepsilon \alpha_e]$$

- The solution is either $\delta \leq q$ or $\delta \geq q$.
- If the change is invalid, the parameter is irrelevant.
- We have to compute the solution of δ for **every parameter** in the Bayesian network.

Complexity

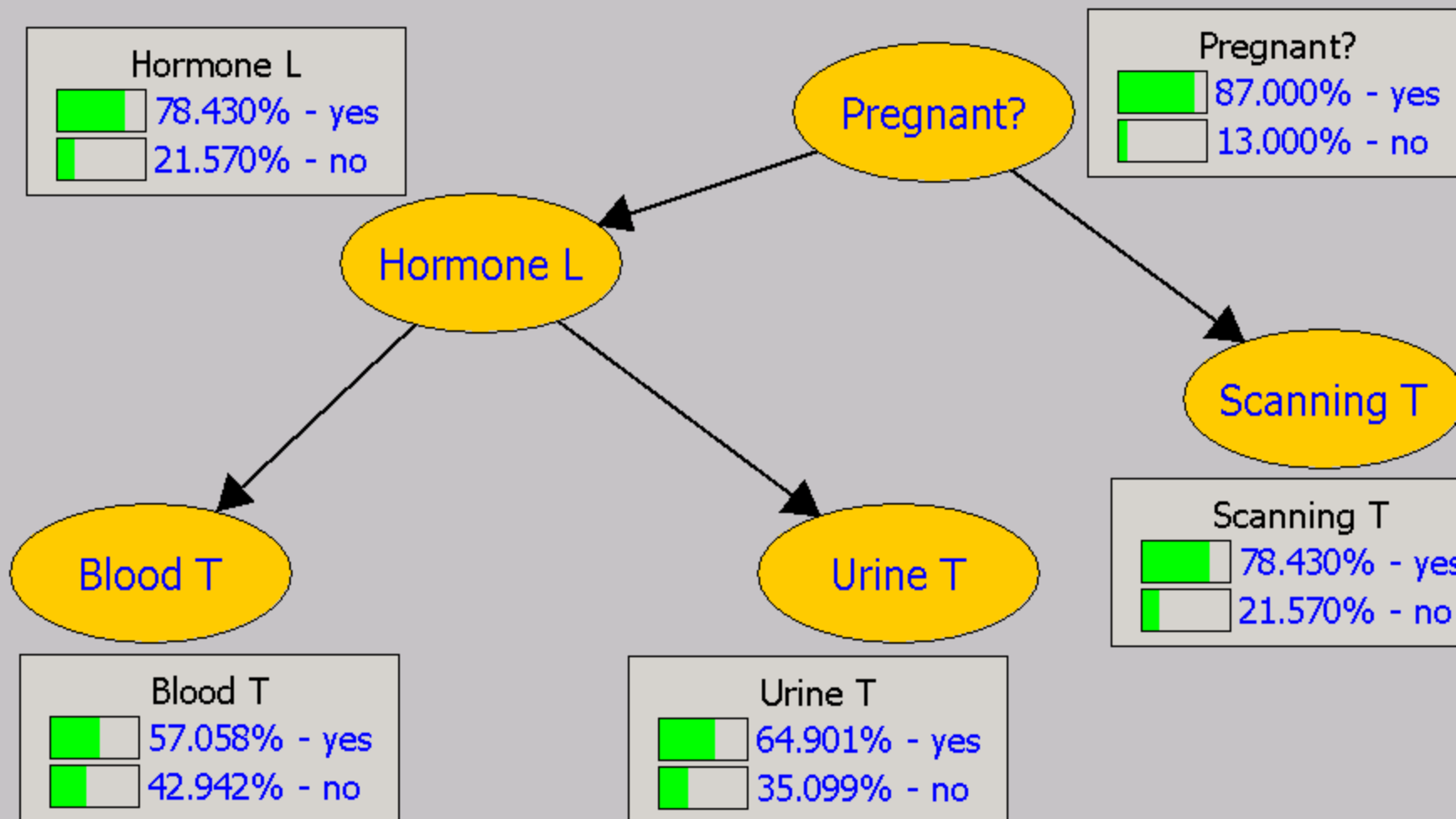
- Linear in the size of AC compilation.
- $O(n 2^w)$, where n is the number of variables, and w is the treewidth.
- Same as performing inference to find $\Pr(e)$,
- the simplest query possible.
- No penalty for sensitivity analysis.

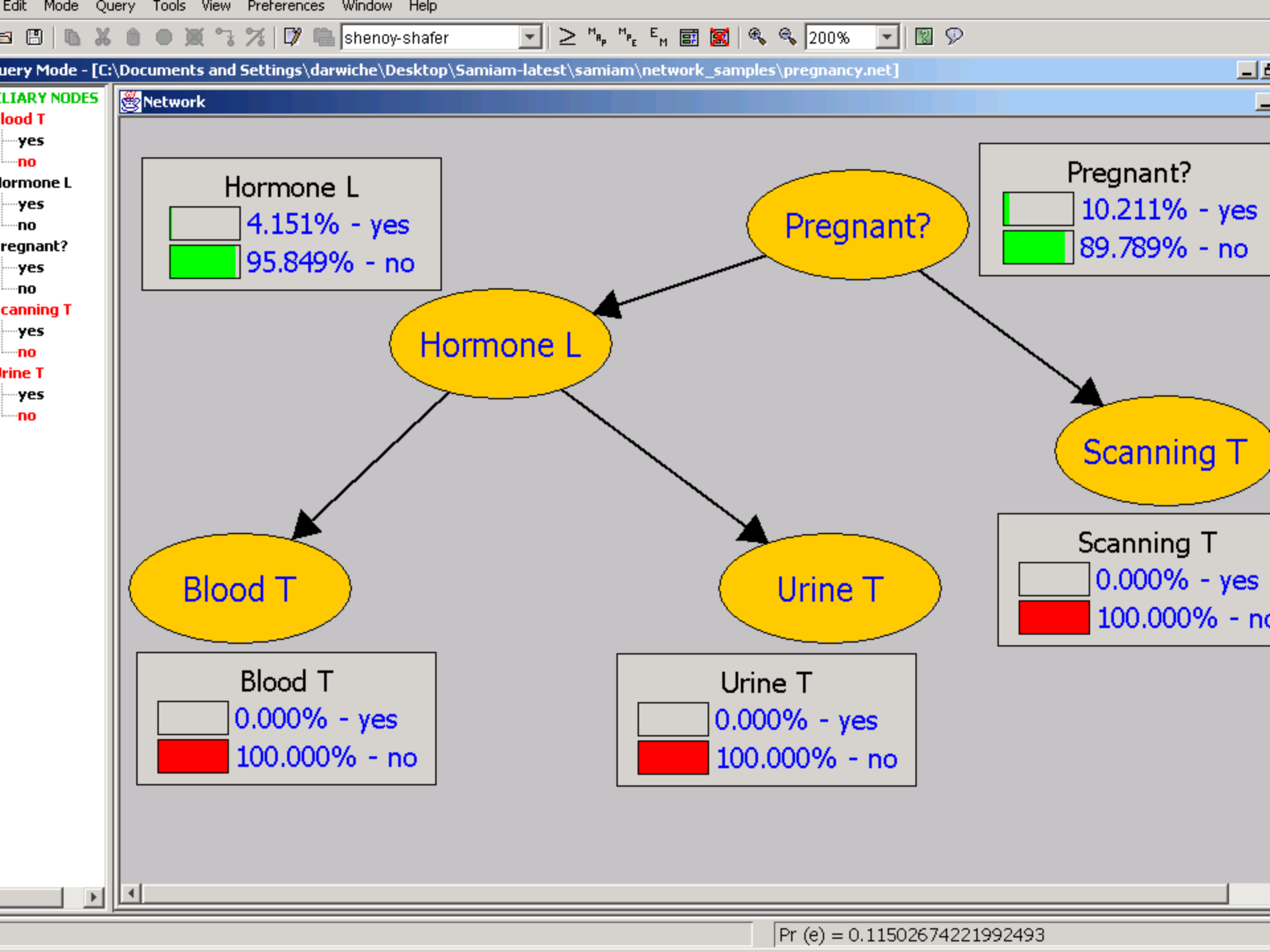


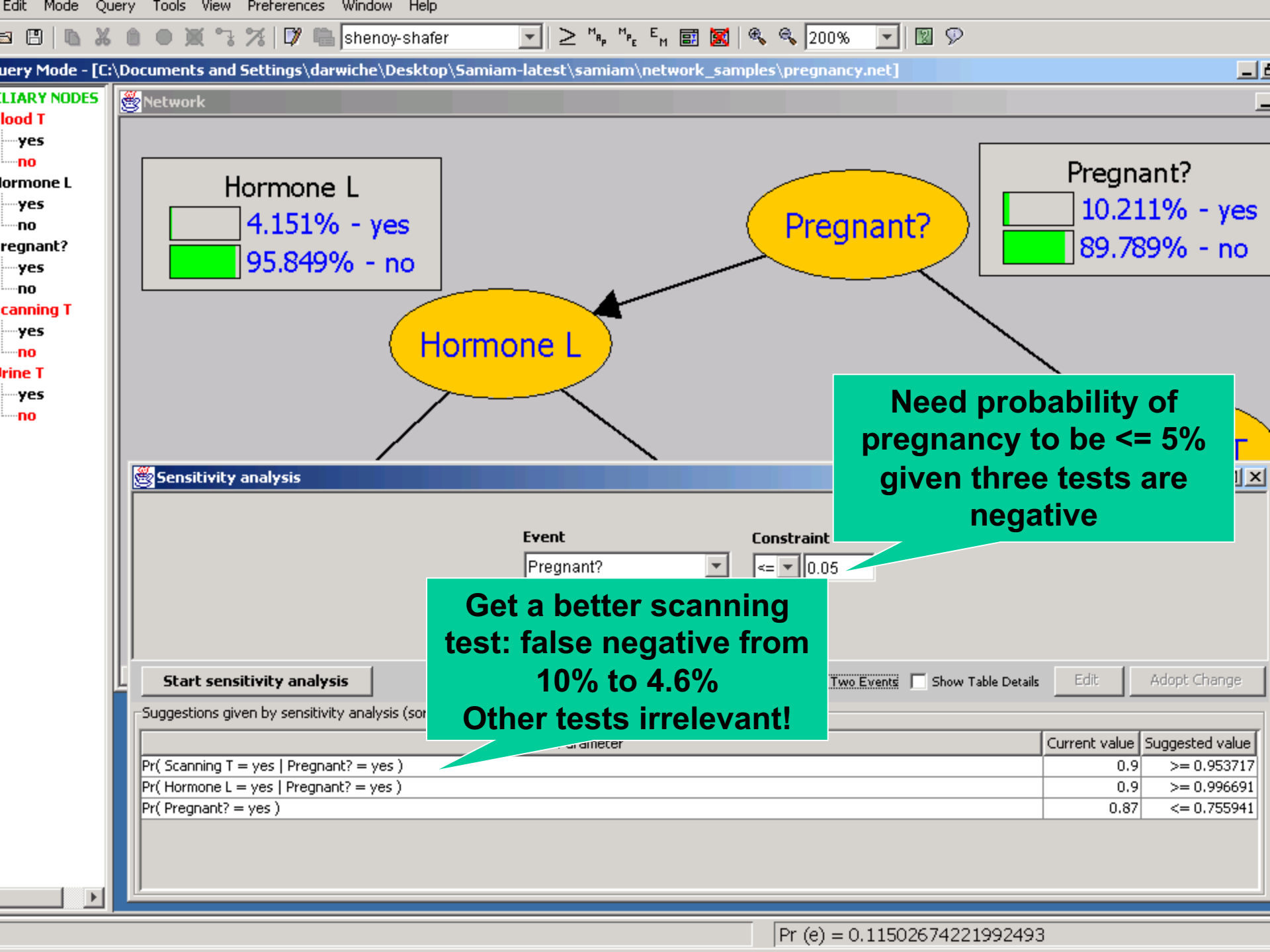
LIBRARY NODES

lood T
yes
no
ormone L
yes
no
regnant?
yes
no
canning T
yes
no
rine T
yes
no

Network







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A Probabilistic Distance Measure

$$D(\text{Pr}, \text{Pr}') \stackrel{\text{def}}{=} \ln \max_w \frac{\text{Pr}'(w)}{\text{Pr}(w)} - \ln \min_w \frac{\text{Pr}'(w)}{\text{Pr}(w)}$$

$D(\text{Pr}, \text{Pr}')$ satisfies the three properties of distance: positiveness, symmetry, and the triangle inequality.

A	B	C	Pr(A,B,C)	Pr' (A,B,C)	Pr' (w) / Pr(w)
a	b	c	0.10	0.20	2.00
a	b	$\bar{c}$	0.20	0.30	1.50
a	$\bar{b}$	c	0.25	0.10	0.40
a	$\bar{b}$	$\bar{c}$	0.05	0.05	1.00
$\bar{a}$	b	c	0.05	0.10	2.00
$\bar{a}$	b	$\bar{c}$	0.10	0.05	0.50
$\bar{a}$	$\bar{b}$	c	0.10	0.10	1.00
$\bar{a}$	$\bar{b}$	$\bar{c}$	0.15	0.10	0.67

max Pr' (w) / Pr(w)
 min Pr' (w) / Pr(w)

$$\begin{aligned}
 D(\text{Pr}, \text{Pr}') &\stackrel{\text{def}}{=} \ln \max_w \frac{\text{Pr}'(w)}{\text{Pr}(w)} - \ln \min_w \frac{\text{Pr}'(w)}{\text{Pr}(w)} \\
 &= \ln 2 - \ln 0.4 = 1.61
 \end{aligned}$$

Significance of Distance Measure

- Let:
 - $\Pr$ and $\Pr'$ be two distributions.
 - α and β be any two events.
 - Odds of $\alpha \mid \beta$ under $\Pr$: $O(\alpha \mid \beta)$.
 - Odds of $\alpha \mid \beta$ under $\Pr'$: $O'(\alpha \mid \beta)$.
- Key result:

$$e^{-D(\Pr, \Pr')} \leq \frac{O'(\alpha \mid \beta)}{O(\alpha \mid \beta)} \leq e^{D(\Pr, \Pr')}$$

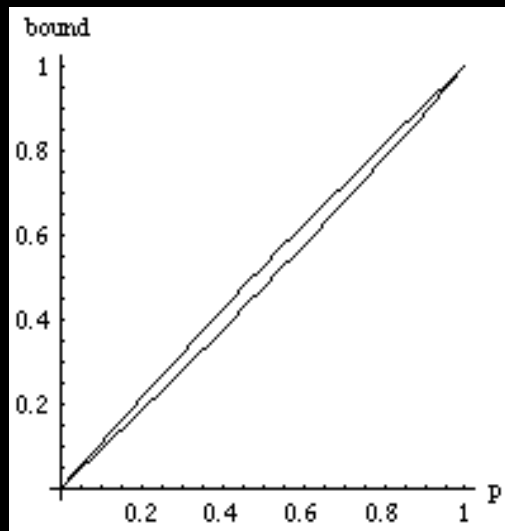
Bounding Belief Change

- Given Pr and Pr' :
 - $p = \text{Pr}(\alpha \mid \beta)$
 - $d = D(\text{Pr}, \text{Pr}')$
- What can we say about $\text{Pr}'(\alpha \mid \beta)$?

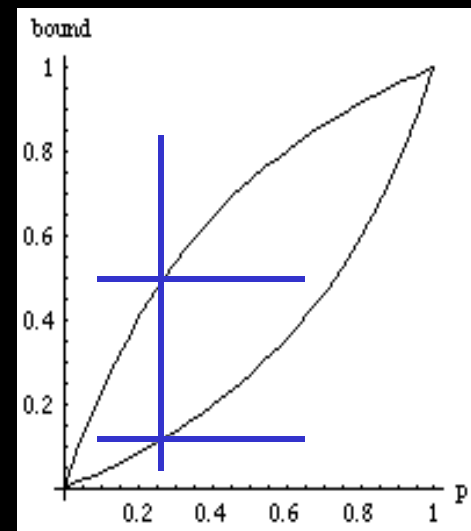
$$\frac{pe^{-d}}{pe^{-d} - p + 1} \leq \text{Pr}'(\alpha \mid \beta) \leq \frac{pe^d}{pe^d - p + 1}$$

Bounding Belief Change

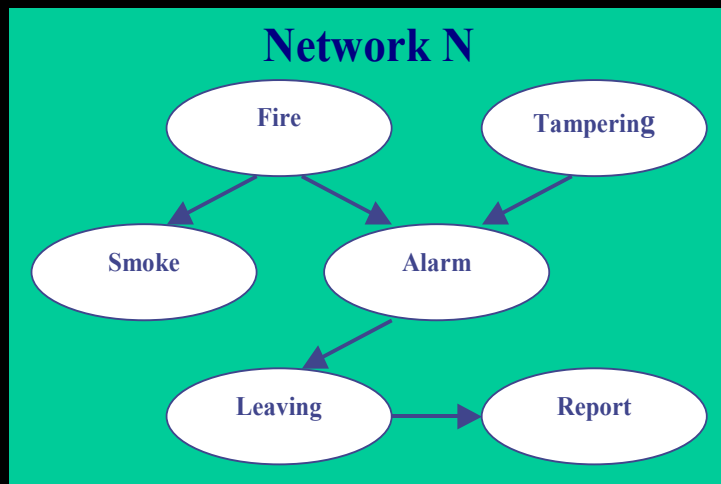
$d = 0.1$



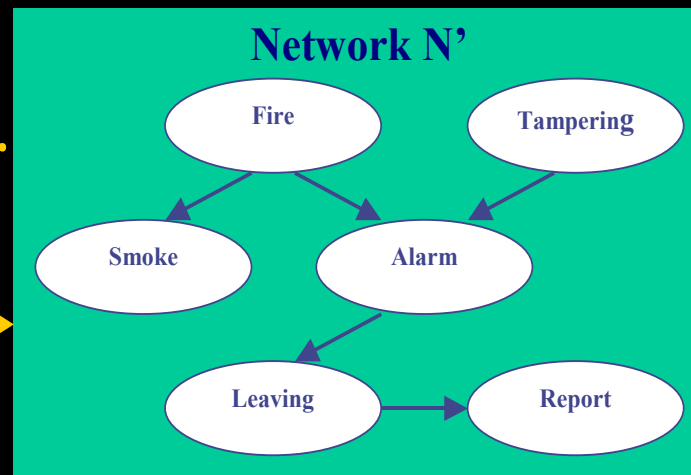
$d = 1$



Applications to Bayesian Networks



**parameter
change**



$$\Pr(\alpha \mid \beta) = p$$

$$\Pr'(\alpha \mid \beta) = ?$$

What is the global impact of this local parameter change?

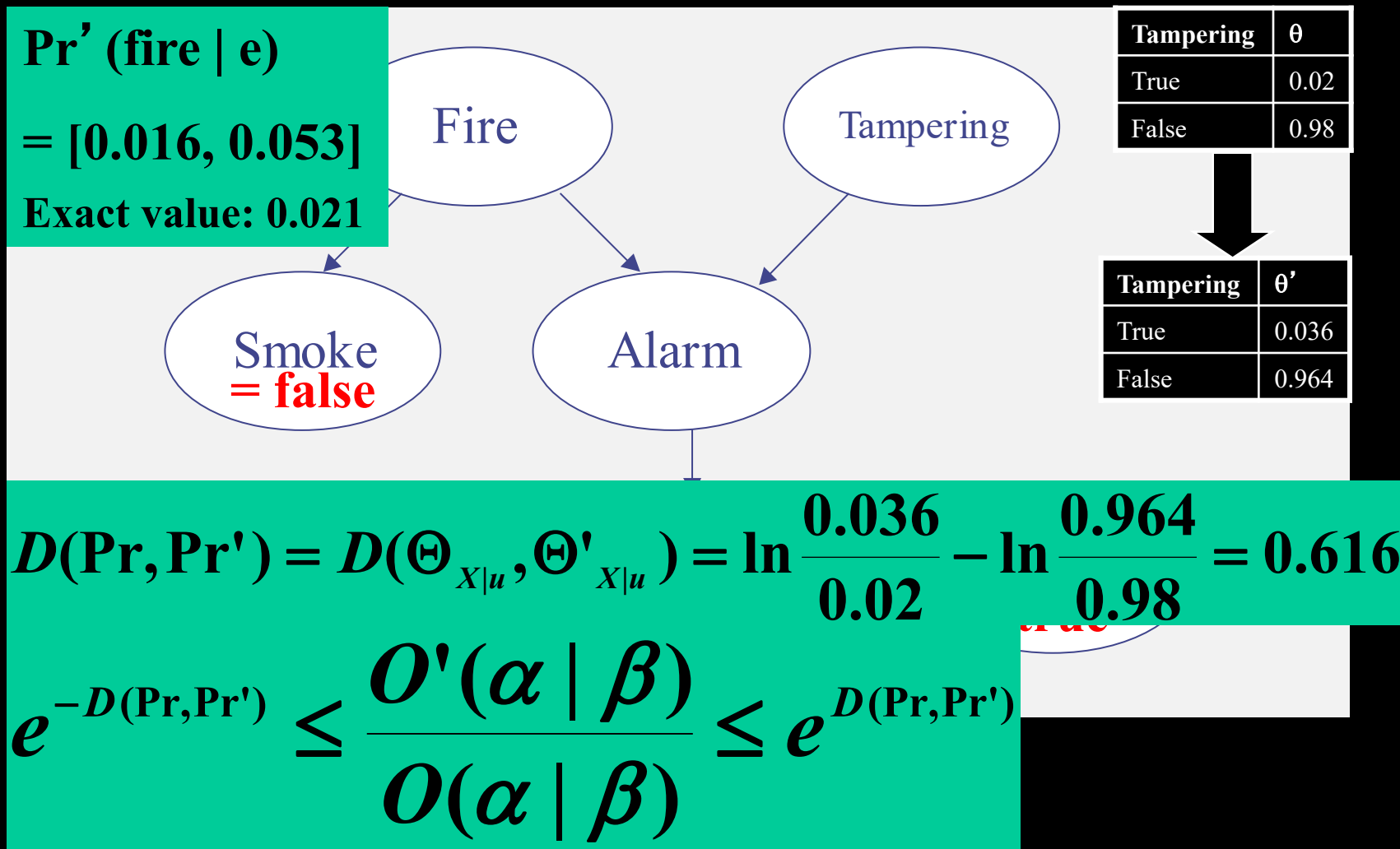
Distance Between Networks

$$D(\text{Pr}, \text{Pr}') = D(\Theta_{X|u}, \Theta'_{X|u})$$

- N' is obtained from N by changing the CPT of X from $\Theta_{X|u}$ to $\Theta'_{X|u}$.
- N and N' induce distributions Pr and Pr' .

$$e^{-D(\Theta_{X|u}, \Theta'_{X|u})} \leq \frac{O'(\alpha | \beta)}{O(\alpha | \beta)} \leq e^{D(\Theta_{X|u}, \Theta'_{X|u})}$$

Bounding Query Change: Example



Measuring Parameter Change

Electricity	θ
Working	0.6
Reduced	0.3
Not working	0.1

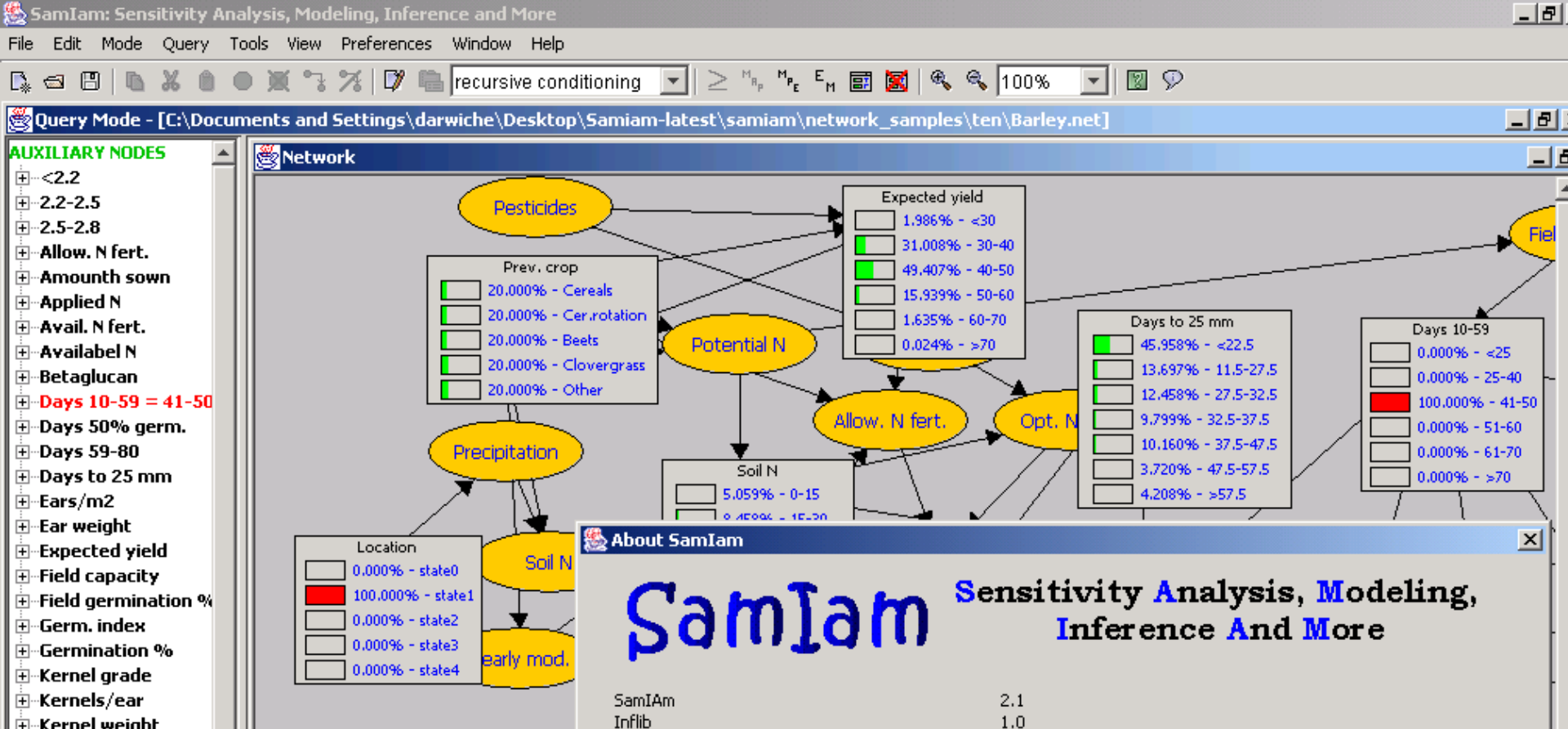
Electricity	θ'
Working	0.725
Reduced	0.225
Not working	0.05

$$D(\Theta_{X|u}, \Theta'_{X|u}) = 0.882$$

Electricity	θ''
Working	0.675
Reduced	0.3
Not working	0.025

$$D(\Theta_{X|u}, \Theta''_{X|u}) = 1.504$$

Which change is bigger?



<http://reasoning.cs.ucla.edu/samiam/>

