## 2.4 Mutable Data

We have seen how abstraction is vital in helping us to cope with the complexity of large systems. Effective programming also requires organizational principles that can guide us in formulating the overall design of a program. In particular, we need strategies to help us structure large systems to be modular, meaning that they divide naturally into coherent parts that can be separately developed and maintained.

One powerful technique for creating modular programs is to incorporate data that may change state over time. In this way, a single data object can represent something that evolves independently of the rest of the program. The behavior of a changing object may be influenced by its history, just like an entity in the world. Adding state to data is a central ingredient of a paradigm called object-oriented programming.

### 2.4.1 The Object Metaphor

In the beginning of this text, we distinguished between functions and data: functions performed operations and data were operated upon. When we included function values among our data, we acknowledged that data too can have behavior. Functions could be manipulated as data, but could also be called to perform computation.

*Objects* combine data values with behavior. Objects represent information, but also *behave* like the things that they represent. The logic of how an object interacts with other objects is bundled along with the information that encodes the object's value. When an object is printed, it knows how to spell itself out in text. If an object is composed of parts, it knows how to reveal those parts on demand. Objects are both information and processes, bundled together to represent the properties, interactions, and behaviors of complex things.

Object behavior is implemented in Python through specialized object syntax and associated terminology, which we can introduce by example. A date is a kind of object.

```
>>> from datetime import date
```

The name `date` is bound to a *class*. As we have seen, a class represents a kind of value. Individual dates are called *instances* of that class. Instances can be *constructed* by calling the class on arguments that characterize the instance.

```
>>> tues = date(2014, 5, 13)
```

While `tues` was constructed from primitive numbers, it behaves like a date. For instance, subtracting it from another date will give a time difference, which we can print.

```
>>> print(date(2014, 5, 19) - tues)
6 days, 0:00:00
```

Objects have *attributes*, which are named values that are part of the object. In Python, like many other programming languages, we use dot notation to designated an attribute of an object.

> <expression> . <name>

Above, the `<expression>` evaluates to an object, and `<name>` is the name of an attribute for that object.

Unlike the names that we have considered so far, these attribute names are not available in the general environment. Instead, attribute names are particular to the object instance preceding the dot.

```
>>> tues.year
2014
```

Objects also have *methods*, which are function-valued attributes. Metaphorically, we say that the object "knows" how to carry out those methods. By implementation, methods are functions that compute their results from both their arguments and their object. For example, The `strftime` method (a classic function name meant to evoke "string format of time") of `tues` takes a single argument that specifies how to display a date (e.g., `%A` means that the day of the week should be spelled out in full).

```
>>> tues.strftime('%A, %B %d')
'Tuesday, May 13'
```

Computing the return value of `strftime` requires two inputs: the string that describes the format of the output and the date information bundled into `tues`. Date-specific logic is applied within this method to yield this result. We never stated that the 13th of May, 2014, was a Tuesday, but knowing the corresponding weekday is part of what it means to be a date. By bundling behavior and information together, this Python object offers us a convincing, self-contained abstraction of a date.

Dates are objects, but numbers, strings, lists, and ranges are all objects as well. They represent values, but also behave in a manner that befits the values they represent. They also have attributes and methods. For instance, strings have an array of methods that facilitate text processing.

```
>>> '1234'.isnumeric()
True
>>> 'rOBERT dE nIRO'.swapcase()
'Robert De Niro'
>>> 'eyes'.upper().endswith('YES')
True
```

In fact, all values in Python are objects. That is, all values have behavior and attributes. They act like the values they represent.

### 2.4.2 Sequence Objects

Instances of primitive built-in values such as numbers are *immutable*. The values themselves cannot change over the course of program execution. Lists on the other hand are *mutable*.

Mutable objects are used to represent values that change over time. A person is the same person from one day to the next, despite having aged, received a haircut, or otherwise changed in some way. Similarly, an object may have changing properties due to *mutating* operations. For example, it is possible to change the contents of a list. Most changes are performed by invoking methods on list objects.

We can introduce many list modification operations through an example that illustrates the history of playing cards (drastically simplified). Comments in the examples describe the effect of each method invocation.

Playing cards were invented in China, perhaps around the 9th century. An early deck had three suits, which corresponded to denominations of money.

```
>>> chinese = ['coin', 'string', 'myriad']  # A list literal
>>> suits = chinese                         # Two names refer to the same list
```

As cards migrated to Europe (perhaps through Egypt), only the suit of coins remained in Spanish decks (*oro*).

```
>>> suits.pop()             # Remove and return the final element
'myriad'
>>> suits.remove('string')  # Remove the first element that equals the argument
```

Three more suits were added (they evolved in name and design over time),

```
>>> suits.append('cup')              # Add an element to the end
>>> suits.extend(['sword', 'club'])  # Add all elements of a sequence to the end
```

and Italians called swords *spades*.

```
>>> suits[2] = 'spade'  # Replace an element
```

giving the suits of a traditional Italian deck of cards.

```
>>> suits
['coin', 'cup', 'spade', 'club']
```

The French variant used today in the U.S. changes the first two suits:

```
>>> suits[0:2] = ['heart', 'diamond']  # Replace a slice
>>> suits
['heart', 'diamond', 'spade', 'club']
```

Methods also exist for inserting, sorting, and reversing lists. All of these mutation operations change the value of the list; they do not create new list objects.

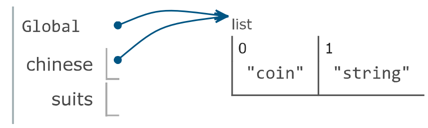
**Sharing and Identity.** Because we have been changing a single list rather than creating new lists, the object bound to the name `chinese` has also changed, because it is the same list object that was bound to `suits`!

```
>>> chinese  # This name co-refers with "suits" to the same changing list
['heart', 'diamond', 'spade', 'club']
```

This behavior is new. Previously, if a name did not appear in a statement, then its value would not be affected by that statement. With mutable data, methods called on one name can affect another name at the same time.

The environment diagram for this example shows how the value bound to `chinese` is changed by statements involving only `suits`. Step through each line of the following example to observe these changes.

```
chinese = ['coin', 'string', 'myriad']
suits = chinese
suits.pop()
suits.remove('string')
suits.append('cup')
suits.extend(['sword', 'club'])
suits[2] = 'spade'
suits[0:2] = ['heart', 'diamond']
```

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< Back Step 4 of 8 Forward >

line that has just executed

next line to execute

Lists can be copied using the `list` constructor function. Changes to one list do not affect another, unless they share structure.

```
>>> nest = list(suits)  # Bind "nest" to a second list with the same elements
>>> nest[0] = suits     # Create a nested list
```

According to this environment, changing the list referenced by `suits` will affect the nested list that is the first element of `nest`, but not the other elements.

```
>>> suits.insert(2, 'Joker')  # Insert an element at index 2, shifting the rest
>>> nest
[['heart', 'diamond', 'Joker', 'spade', 'club'], 'diamond', 'spade', 'club']
```

And likewise, undoing this change in the first element of `nest` will change `suit` as well.

```
>>> nest[0].pop(2)
'Joker'
>>> suits
['heart', 'diamond', 'spade', 'club']
```

Stepping through this example line by line will show the representation of a nested list.

```
suits = ['heart', 'diamond', 'spade', 'club']
nest = list(suits)
nest[0] = suits
suits.insert(2, 'Joker')
joke = nest[0].pop(2)
```

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< Back Step 1 of 5 Forward >

line that has just executed

next line to execute

Because two lists may have the same contents but in fact be different lists, we require a means to test whether two objects are the same. Python includes two comparison operators, called `is` and `is not`, that test whether two expressions in fact evaluate to the identical object. Two objects are identical if they are equal in their current value, and any change to one will always be reflected in the other. Identity is a stronger condition than equality.

```
>>> suits is nest[0]
True
>>> suits is ['heart', 'diamond', 'spade', 'club']
False
>>> suits == ['heart', 'diamond', 'spade', 'club']
True
```

The final two comparisons illustrate the difference between `is` and `==`. The former checks for identity, while the latter checks for the equality of contents.

**List comprehensions.** A list comprehension always creates a new list. For example, the `unicodedata` module tracks the official names of every character in the Unicode alphabet. We can look up the characters corresponding to names, including those for card suits.

```
>>> from unicodedata import lookup
>>> [lookup('WHITE ' + s.upper() + ' SUIT') for s in suits]
['♡', '♢', '♤', '♧']
```

This resulting list does not share any of its contents with `suits`, and evaluating the list comprehension does not modify the `suits` list.

You can read more about the Unicode standard for representing text in the Unicode section of Dive into Python 3.

**Tuples.** A tuple, an instance of the built-in `tuple` type, is an immutable sequence. Tuples are created using a tuple literal that separates element expressions by commas. Parentheses are optional but used commonly in practice. Any objects can be placed within tuples.

```
>>> 1, 2 + 3
(1, 5)
>>> ("the", 1, ("and", "only"))
('the', 1, ('and', 'only'))
>>> type( (10, 20) )
<class 'tuple'>
```

Empty and one-element tuples have special literal syntax.

```
>>> ()    # 0 elements
()
>>> (10,) # 1 element
(10,)
```

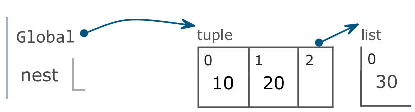
Like lists, tuples have a finite length and support element selection. They also have a few methods that are also available for lists, such as `count` and `index`.

```
>>> code = ("up", "up", "down", "down") + ("left", "right") * 2
>>> len(code)
8
>>> code[3]
'down'
>>> code.count("down")
2
>>> code.index("left")
4
```

However, the methods for manipulating the contents of a list are not available for tuples because tuples are immutable.

While it is not possible to change which elements are in a tuple, it is possible to change the value of a mutable element contained within a tuple.

```
nest = (10, 20, [30, 40])
nest[2].pop()
```

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< Back End Forward >

line that has just executed
next line to execute

Tuples are used implicitly in multiple assignment. An assignment of two values to two names creates a two-element tuple and then unpacks it.

### 2.4.3 Dictionaries

Dictionaries are Python's built-in data type for storing and manipulating correspondence relationships. A dictionary contains key-value pairs, where both the keys and values are objects. The purpose of a dictionary is to provide an abstraction for storing and retrieving values that are indexed not by consecutive integers, but by descriptive keys.

Strings commonly serve as keys, because strings are our conventional representation for names of things. This dictionary literal gives the values of various Roman numerals.

```
>>> numerals = {'I': 1.0, 'V': 5, 'X': 10}
```

Looking up values by their keys uses the element selection operator that we previously applied to sequences.

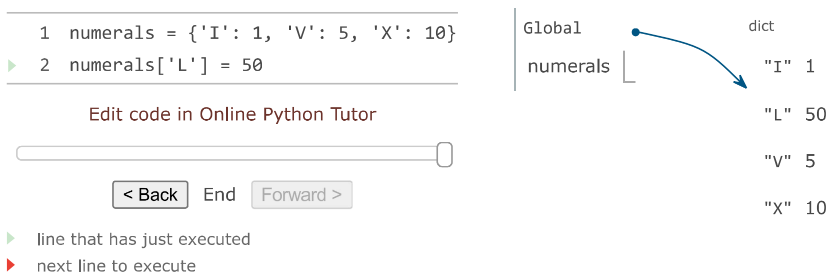
```
>>> numerals['X']
10
```

A dictionary can have at most one value for each key. Adding new key-value pairs and changing the existing value for a key can both be achieved with assignment statements.

```
>>> numerals['I'] = 1
>>> numerals['L'] = 50
>>> numerals
{'I': 1, 'X': 10, 'L': 50, 'V': 5}
```

Notice that `'L'` was not added to the end of the output above. Dictionaries are unordered collections of key-value pairs. When we print a dictionary, the keys and values are rendered in some order, but as users of the language we cannot predict what that order will be. The order may change when running a program multiple times.

Dictionaries can appear in environment diagrams as well.




The dictionary type also supports various methods of iterating over the contents of the dictionary as a whole. The methods `keys`, `values`, and `items` all return iterable values.

```
>>> sum(numerals.values())
66
```

A list of key-value pairs can be converted into a dictionary by calling the `dict` constructor function.

```
>>> dict([(3, 9), (4, 16), (5, 25)])
{3: 9, 4: 16, 5: 25}
```

Dictionaries do have some restrictions:

- A key of a dictionary cannot be or contain a mutable value.
- There can be at most one value for a given key.

This first restriction is tied to the underlying implementation of dictionaries in Python. The details of this implementation are not a topic of this text. Intuitively, consider that the key tells Python where to find that key-value pair in memory; if the key changes, the location of the pair may be lost. Tuples are commonly used for keys in dictionaries because lists cannot be used.

The second restriction is a consequence of the dictionary abstraction, which is designed to store and retrieve values for keys. We can only retrieve *the* value for a key if at most one such value exists in the dictionary.

A useful method implemented by dictionaries is `get`, which returns either the value for a key, if the key is present, or a default value. The arguments to `get` are the key and the default value.

```
>>> numerals.get('A', 0)
0
>>> numerals.get('V', 0)
5
```

Dictionaries also have a comprehension syntax analogous to those of lists. A key expression and a value expression are separated by a colon. Evaluating a dictionary comprehension creates a new dictionary object.

```
>>> {x: x*x for x in range(3,6)}
{3: 9, 4: 16, 5: 25}
```

### 2.4.4 Local State

Lists and dictionaries have *local state*: they are changing values that have some particular contents at any point in the execution of a program. The word "state" implies an evolving process in which that state may change.

Functions can also have local state. For instance, let us define a function that models the process of withdrawing money from a bank account. We will create a function called `withdraw`, which takes as its argument an amount to be withdrawn. If there is enough money in the account to accommodate the withdrawal, then `withdraw` will return the balance remaining after the withdrawal. Otherwise, `withdraw` will return the message `'Insufficient funds'`. For example, if we begin with $100 in the account, we would like to obtain the following sequence of return values by calling withdraw:

```
>>> withdraw(25)
75
>>> withdraw(25)
50
>>> withdraw(60)
'Insufficient funds'
>>> withdraw(15)
35
```

Above, the expression `withdraw(25)`, evaluated twice, yields different values. Thus, this user-defined function is non-pure. Calling the function not only returns a value, but also has the side effect of changing the function in some way, so that the next call with the same argument will return a different result. This side effect is a result of `withdraw` making a change to a name-value binding outside of the current frame.

For `withdraw` to make sense, it must be created with an initial account balance. The function `make_withdraw` is a higher-order function that takes a starting balance as an argument. The function `withdraw` is its return value.

```
>>> withdraw = make_withdraw(100)
```

An implementation of `make_withdraw` requires a new kind of statement: a `nonlocal` statement. When we call `make_withdraw`, we bind the name `balance` to the initial amount. We then define and return a local function, `withdraw`, which updates and returns the value of `balance` when called.

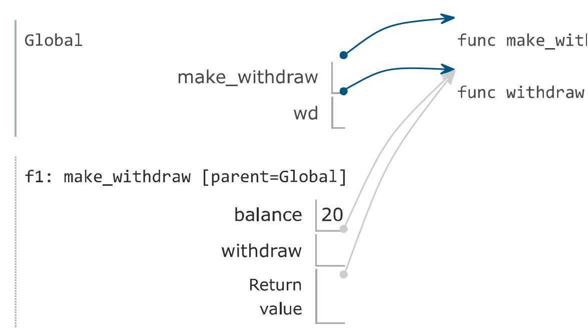
```
>>> def make_withdraw(balance):
        """Return a withdraw function that draws down balance with each call."""
        def withdraw(amount):
            nonlocal balance                 # Declare the name "balance" nonlocal
            if amount > balance:
                return 'Insufficient funds'
            balance = balance - amount       # Re-bind the existing balance name
            return balance
        return withdraw
```

The `nonlocal` statement declares that whenever we change the binding of the name `balance`, the binding is changed in the first frame in which `balance` is already bound. Recall that without the `nonlocal` statement, an assignment statement would always bind a name in the first frame of the current environment. The `nonlocal` statement indicates that the name appears somewhere in the environment other than the first (local) frame or the last (global) frame.

The following environment diagrams illustrate the effects of multiple calls to a function created by `make_withdraw`.

```
def make_withdraw(balance):
    def withdraw(amount):
        nonlocal balance
        if amount > balance:
            return 'Insufficient funds'
        balance = balance - amount
        return balance
    return withdraw

wd = make_withdraw(20)
wd(5)
wd(3)
```

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< Back Step 6 of 15 Forward >

line that has just executed

next line to execute

The first def statement has the usual effect: it creates a new user-defined function and binds the name `make_withdraw` to that function in the global frame. The subsequent call to `make_withdraw` creates and returns a locally defined function `withdraw`. The name `balance` is bound in the parent frame of this function. Crucially, there will only be this single binding for the name `balance` throughout the rest of this example.

Next, we evaluate an expression that calls this function, bound to the name `wd`, on an amount 5. The body of `withdraw` is executed in a new environment that extends the environment in which `withdraw` was defined. Tracing the effect of evaluating `withdraw` illustrates the effect of a `nonlocal` statement in Python: a name outside of the first local frame can be changed by an assignment statement.

```
def make_withdraw(balance):
    def withdraw(amount):
        nonlocal balance
        if amount > balance:
            return 'Insufficient funds'
        balance = balance - amount
        return balance
    return withdraw

wd = make_withdraw(20)
wd(5)
wd(3)
```

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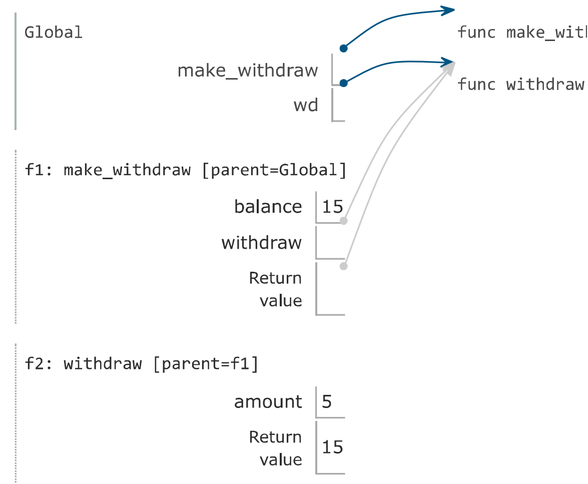
< Back Step 11 of 15 Forward >

line that has just executed
next line to execute




The `nonlocal` statement changes all of the remaining assignment statements in the definition of `withdraw`. After executing `nonlocal balance`, any assignment statement with `balance` on the left-hand side of `=` will not bind `balance` in the first frame of the current environment. Instead, it will find the first frame in which `balance` was already defined and re-bind the name in that frame. If `balance` has not previously been bound to a value, then the `nonlocal` statement will give an error.

By virtue of changing the binding for `balance`, we have changed the `withdraw` function as well. The next time it is called, the name `balance` will evaluate to 15 instead of 20. Hence, when we call `withdraw` a second time, we see that its return value is 12 and not 17. The change to `balance` from the first call affects the result of the second call.

```
def make_withdraw(balance):
    def withdraw(amount):
        nonlocal balance
        if amount > balance:
            return 'Insufficient funds'
        balance = balance - amount
        return balance
    return withdraw

wd = make_withdraw(20)
wd(5)
wd(3)
```

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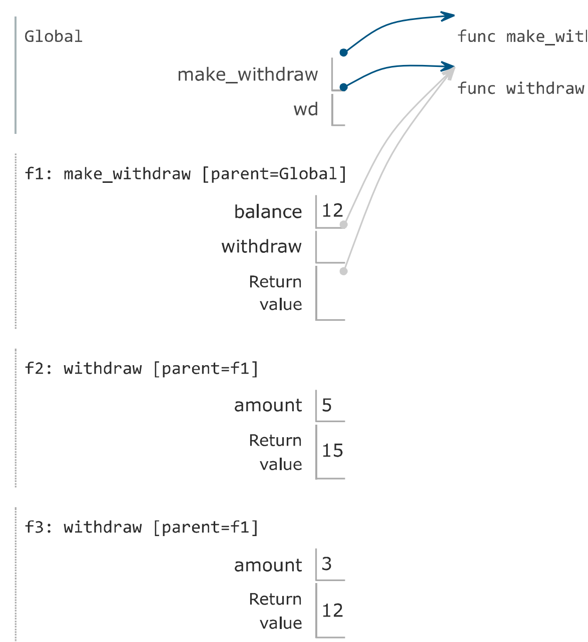
< Back End Forward >

line that has just executed
next line to execute




The second call to `withdraw` does create a second local frame, as usual. However, both `withdraw` frames have the same parent. That is, they both extend the environment for `make_withdraw`, which contains the binding for `balance`. Hence, they share that particular name binding. Calling `withdraw` has the side effect of altering the environment that will be extended by future calls to `withdraw`. The `nonlocal` statement allows `withdraw` to change a name binding in the `make_withdraw` frame.

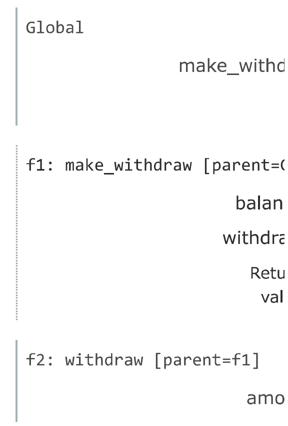
Ever since we first encountered nested `def` statements, we have observed that a locally defined function can look up names outside of its local frames. No `nonlocal` statement is required to *access* a non-local name. By contrast, only after a `nonlocal` statement can a function *change* the binding of names in these frames.

By introducing `nonlocal` statements, we have created a dual role for assignment statements. Either they change local bindings, or they change nonlocal bindings. In fact, assignment statements already had a dual role: they either created new bindings or re-bound existing names. Assignment can also change the contents of lists and dictionaries. The many roles of Python assignment can obscure the effects of executing an assignment statement. It is up to you as a programmer to document your code clearly so that the effects of assignment can be understood by others.

**Python Particulars.** This pattern of non-local assignment is a general feature of programming languages with higher-order functions and lexical scope. Most other languages do not require a `nonlocal` statement at all. Instead, non-local assignment is often the default behavior of assignment statements.

Python also has an unusual restriction regarding the lookup of names: within the body of a function, all instances of a name must refer to the same frame. As a result, Python cannot look up the value of a name in a non-local frame, then bind that same name in the local frame, because the same name would be accessed in two different frames in the same function. This restriction allows Python to pre-compute which frame contains each name before executing the body of a function. When this restriction is violated, a confusing error message results. To demonstrate, the `make_withdraw` example is repeated below with the `nonlocal` statement removed.

```
1  def make_withdraw(balance):
2      def withdraw(amount):
3          if amount > balance:
4              return 'Insufficient funds'
5          balance = balance - amount
6          return balance
7      return withdraw
8
9  wd = make_withdraw(20)
10 wd(5)
```

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< Back Step 8 of 9 Forward >

UnboundLocalError: local variable 'balance' referenced before assignment

line that has just executed

next line to execute

This `UnboundLocalError` appears because `balance` is assigned locally in line 5, and so Python assumes that all references to `balance` must appear in the local frame as well. This error occurs *before* line 5 is ever executed, implying that Python has considered line 5 in some way before executing line 3. As we study interpreter design, we will see that pre-computing facts about a function body before executing it is quite common. In this case, Python's pre-processing restricted the frame in which `balance` could appear, and thus prevented the name from being found. Adding a `nonlocal` statement corrects this error. The `nonlocal` statement did not exist in Python 2.

### 2.4.5 The Benefits of Non-Local Assignment

Non-local assignment is an important step on our path to viewing a program as a collection of independent and autonomous *objects*, which interact with each other but each manage their own internal state.

In particular, non-local assignment has given us the ability to maintain some state that is local to a function, but evolves over successive calls to that function. The `balance` associated with a particular withdraw function is shared among all calls to that function. However, the binding for balance associated with an instance of withdraw is inaccessible to the rest of the program. Only `wd` is associated with the frame for `make_withdraw` in which it was defined. If `make_withdraw` is called again, then it will create a separate frame with a separate binding for `balance`.

We can extend our example to illustrate this point. A second call to `make_withdraw` returns a second `withdraw` function that has a different parent. We bind this second function to the name `wd2` in the global frame.

```
def make_withdraw(balance):
    def withdraw(amount):
        nonlocal balance
        if amount > balance:
            return 'Insufficient funds'
        balance = balance - amount
        return balance
    return withdraw

wd = make_withdraw(20)
wd2 = make_withdraw(7)
wd2(6)
wd(8)
```

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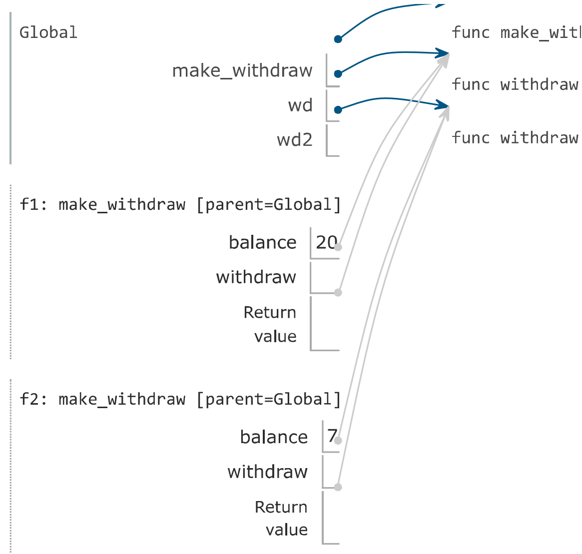
< Back Step 10 of 19 Forward >

line that has just executed
next line to execute




Now, we see that there are in fact two bindings for the name `balance` in two different frames, and each `withdraw` function has a different parent. The name `wd` is bound to a function with a balance of 20, while `wd2` is bound to a different function with a balance of 7.

Calling `wd2` changes the binding of its non-local `balance` name, but does not affect the function bound to the name `withdraw`. A future call to `wd` is unaffected by the changing balance of `wd2`; its balance is still 20.

```
def make_withdraw(balance):
    def withdraw(amount):
        nonlocal balance
        if amount > balance:
            return 'Insufficient funds'
        balance = balance - amount
        return balance
    return withdraw

wd = make_withdraw(20)
wd2 = make_withdraw(7)
wd2(6)
wd(8)
```

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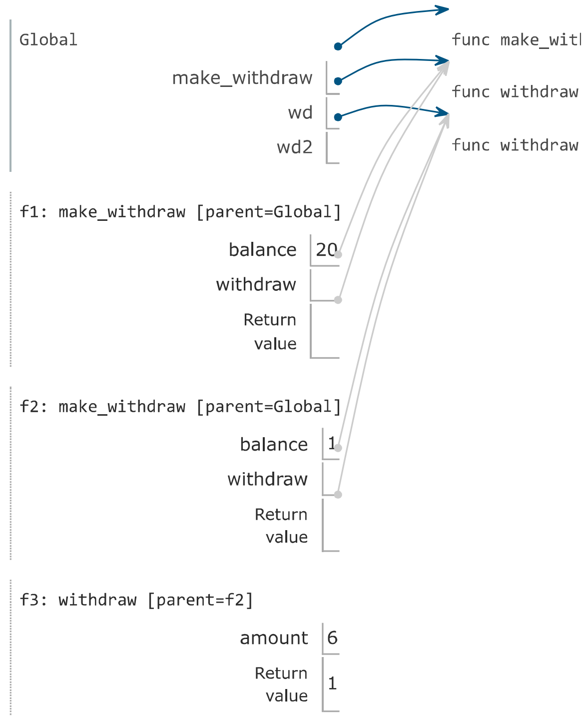
< Back Step 15 of 19 Forward >

line that has just executed
next line to execute




In this way, each instance of `withdraw` maintains its own balance state, but that state is inaccessible to any other function in the program. Viewing this situation at a higher level, we have created an abstraction of a bank account that manages its own internals but behaves in a way that models accounts in the world: it changes over time based on its own history of withdrawal requests.

### 2.4.6 The Cost of Non-Local Assignment

Our environment model of computation cleanly extends to explain the effects of non-local assignment. However, non-local assignment introduces some important nuances in the way we think about names and values.

Previously, our values did not change; only our names and bindings changed. When two names `a` and `b` were both bound to the value 4, it did not matter whether they were bound to the same 4 or different

4's. As far as we could tell, there was only one 4 object that never changed.

However, functions with state do not behave this way. When two names `wd` and `wd2` are both bound to a `withdraw` function, it *does* matter whether they are bound to the same function or different instances of that function. Consider the following example, which contrasts the one we just analyzed. In this case, calling the function named by `wd2` did change the value of the function named by `wd`, because both names refer to the same function.

```
def make_withdraw(balance):
    def withdraw(amount):
        nonlocal balance
        if amount > balance:
            return 'Insufficient funds'
        balance = balance - amount
        return balance
    return withdraw

wd = make_withdraw(12)
wd2 = wd
wd2(1)
wd(1)
```

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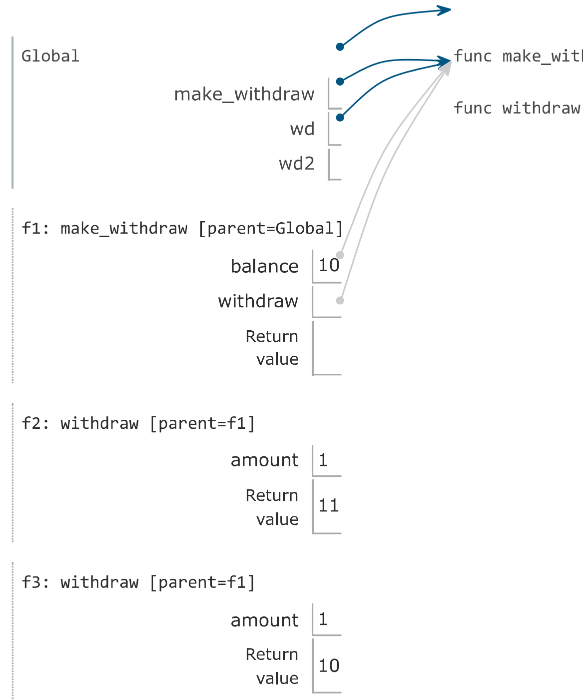
< Back End Forward >

line that has just executed
next line to execute




It is not unusual for two names to co-refer to the same value in the world, and so it is in our programs. But, as values change over time, we must be very careful to understand the effect of a change on other names that might refer to those values.

The key to correctly analyzing code with non-local assignment is to remember that only function calls can introduce new frames. Assignment statements always change bindings in existing frames. In this case, unless `make_withdraw` is called twice, there can be only one binding for `balance`.

**Sameness and change.** These subtleties arise because, by introducing non-pure functions that change the non-local environment, we have changed the nature of expressions. An expression that contains only pure function calls is *referentially transparent*; its value does not change if we substitute one of its subexpression with the value of that subexpression.

Re-binding operations violate the conditions of referential transparency because they do more than return a value; they change the environment. When we introduce arbitrary re-binding, we encounter a thorny epistemological issue: what it means for two values to be the same. In our environment model of computation, two separately defined functions are not the same, because changes to one may not be reflected in the other.

In general, so long as we never modify data objects, we can regard a compound data object to be precisely the totality of its pieces. For example, a rational number is determined by giving its numerator and its denominator. But this view is no longer valid in the presence of change, where a compound data object has an "identity" that is something different from the pieces of which it is composed. A bank account is still "the same" bank account even if we change the balance by making a withdrawal; conversely, we could have two bank accounts that happen to have the same balance, but are different objects.

Despite the complications it introduces, non-local assignment is a powerful tool for creating modular programs. Different parts of a program, which correspond to different environment frames, can evolve separately throughout program execution. Moreover, using functions with local state, we are able to implement mutable data types. In fact, we can implement abstract data types that are equivalent to the built-in `list` and `dict` types introduced above.

### 2.4.7 Implementing Lists and Dictionaries

The Python language does not give us access to the implementation of lists, only to the sequence abstraction and mutation methods built into the language. To understand how a mutable list could be represented using functions with local state, we will now develop an implementation of a mutable linked list.

We will represent a mutable linked list by a function that has a linked list as its local state. Lists need to have an identity, like any mutable value. In particular, we cannot use `None` to represent an empty mutable list, because two empty lists are not identical values (e.g., appending to one does not append to the other), but `None is None`. On the other hand, two different functions that each have `empty` as their local state will suffice to distinguish two empty lists.

If a mutable linked list is a function, what arguments does it take? The answer exhibits a general pattern in programming: the function is a dispatch function and its arguments are first a message, followed by additional arguments to parameterize that method. This message is a string naming what the function should do. Dispatch functions are effectively many functions in one: the message determines the behavior of the function, and the additional arguments are used in that behavior.

Our mutable list will respond to five different messages: `len`, `getitem`, `push_first`, `pop_first`, and `str`. The first two implement the behaviors of the sequence abstraction. The next two add or remove the first element of the list. The final message returns a string representation of the whole linked list.

```
>>> def mutable_link():
        """Return a functional implementation of a mutable linked list."""
        contents = empty
        def dispatch(message, value=None):
            nonlocal contents
            if message == 'len':
                return len_link(contents)
            elif message == 'getitem':
                return getitem_link(contents, value)
            elif message == 'push_first':
                contents = link(value, contents)
            elif message == 'pop_first':
                f = first(contents)
                contents = rest(contents)
                return f
            elif message == 'str':
                return join_link(contents, ", ")
        return dispatch
```

We can also add a convenience function to construct a functionally implemented linked list from any built-in sequence, simply by adding each element in reverse order.

```
>>> def to_mutable_link(source):
        """Return a functional list with the same contents as source."""
        s = mutable_link()
        for element in reversed(source):
            s('push_first', element)
        return s
```

In the definition above, the function `reversed` takes and returns an iterable value; it is another example of a function that processes sequences.

At this point, we can construct a functionally implemented mutable linked lists. Note that the linked list itself is a function.

```
>>> s = to_mutable_link(suits)
>>> type(s)
<class 'function'>
>>> print(s('str'))
heart, diamond, spade, club
```

In addition, we can pass messages to the list `s` that change its contents, for instance removing the first element.

```
>>> s('pop_first')
'heart'
>>> print(s('str'))
diamond, spade, club
```

In principle, the operations `push_first` and `pop_first` suffice to make arbitrary changes to a list. We can always empty out the list entirely and then replace its old contents with the desired result.

**Message passing.** Given some time, we could implement the many useful mutation operations of Python lists, such as `extend` and `insert`. We would have a choice: we could implement them all as functions, which use the existing messages `pop_first` and `push_first` to make all changes. Alternatively, we could add additional `elif` clauses to the body of `dispatch`, each checking for a message (e.g., `'extend'`) and applying the appropriate change to `contents` directly.

This second approach, which encapsulates the logic for all operations on a data value within one function that responds to different messages, is a discipline called message passing. A program that uses message passing defines dispatch functions, each of which may have local state, and organizes computation by passing "messages" as the first argument to those functions. The messages are strings that correspond to particular behaviors.

**Implementing Dictionaries.** We can also implement a value with similar behavior to a dictionary. In this case, we use a list of key-value pairs to store the contents of the dictionary. Each pair is a two-element list.

```
>>> def dictionary():
        """Return a functional implementation of a dictionary."""
        records = []
        def getitem(key):
            matches = [r for r in records if r[0] == key]
            if len(matches) == 1:
                key, value = matches[0]
                return value
        def setitem(key, value):
            nonlocal records
            non_matches = [r for r in records if r[0] != key]
            records = non_matches + [[key, value]]
        def dispatch(message, key=None, value=None):
            if message == 'getitem':
                return getitem(key)
            elif message == 'setitem':
                setitem(key, value)
        return dispatch
```

Again, we use the message passing method to organize our implementation. We have supported two messages: `getitem` and `setitem`. To insert a value for a key, we filter out any existing records with the given key, then add one. In this way, we are assured that each key appears only once in records. To look up a value for a key, we filter for the record that matches the given key. We can now use our implementation to store and retrieve values.

```
>>> d = dictionary()
>>> d('setitem', 3, 9)
>>> d('setitem', 4, 16)
>>> d('getitem', 3)
9
>>> d('getitem', 4)
16
```

This implementation of a dictionary is *not* optimized for fast record lookup, because each call must filter through all records. The built-in dictionary type is considerably more efficient. The way in which it is implemented is beyond the scope of this text.

### 2.4.8 Dispatch Dictionaries

The dispatch function is a general method for implementing a message passing interface for abstract data. To implement message dispatch, we have thus far used conditional statements to compare the message string to a fixed set of known messages.

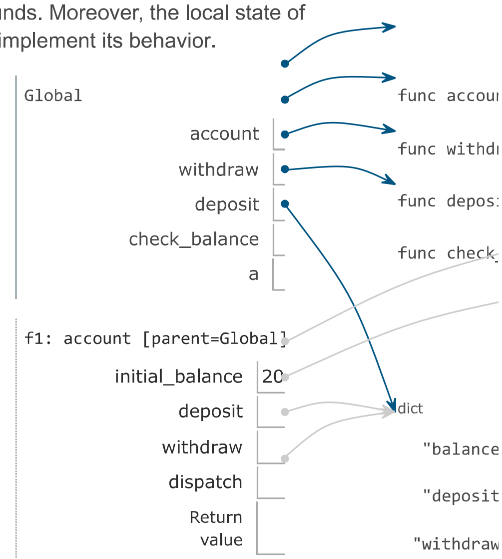
The built-in dictionary data type provides a general method for looking up a value for a key. Instead of using conditionals to implement dispatching, we can use dictionaries with string keys.

The mutable `account` data type below is implemented as a dictionary. It has a constructor `account` and selector `check_balance`, as well as functions to `deposit` or `withdraw` funds. Moreover, the local state of the account is stored in the dictionary alongside the functions that implement its behavior.

```
        dispatch['balance'] -= amount
        return dispatch['balance']
    dispatch = {'deposit':   deposit,
                'withdraw':  withdraw,
                'balance':   initial_balance}
    return dispatch

def withdraw(account, amount):
    return account['withdraw'](amount)
def deposit(account, amount):
    return account['deposit'](amount)
def check_balance(account):
    return account['balance']

a = account(20)
deposit(a, 5)
withdraw(a, 17)
check_balance(a)
```

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< Back Step 13 of 28 Forward >

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The name dispatch within the body of the account constructor is bound to a dictionary that contains the messages accepted by an account as keys. The *balance* is a number, while the messages *deposit* and *withdraw* are bound to functions. These functions have access to the dispatch dictionary, and so they can read and change the balance. By storing the balance in the dispatch dictionary rather than in the account frame directly, we avoid the need for nonlocal statements in deposit and withdraw.

The operators += and -= are shorthand in Python (and many other languages) for combined lookup and re-assignment. The last two lines below are equivalent.

```
>>> a = 2
>>> a = a + 1
>>> a += 1
```

### 2.4.9 Propagating Constraints

Mutable data allows us to simulate systems with change, but also allows us to build new kinds of abstractions. In this extended example, we combine nonlocal assignment, lists, and dictionaries to build a *constraint-based system* that supports computation in multiple directions. Expressing programs as constraints is a type of *declarative programming*, in which a programmer declares the structure of a problem to be solved, but abstracts away the details of exactly how the solution to the problem is computed.

Computer programs are traditionally organized as one-directional computations, which perform operations on pre-specified arguments to produce desired outputs. On the other hand, we often want to model systems in terms of relations among quantities. For example, we previously considered the ideal gas law, which relates the pressure (p), volume (v), quantity (n), and temperature (t) of an ideal gas via Boltzmann's constant (k):

```
p * v = n * k * t
```

Such an equation is not one-directional. Given any four of the quantities, we can use this equation to compute the fifth. Yet translating the equation into a traditional computer language would force us to choose one of the quantities to be computed in terms of the other four. Thus, a function for computing the pressure could not be used to compute the temperature, even though the computations of both quantities arise from the same equation.

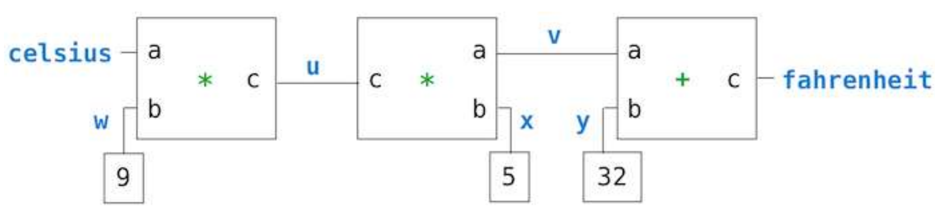
In this section, we sketch the design of a general model of linear relationships. We define primitive constraints that hold between quantities, such as an adder(a, b, c) constraint that enforces the mathematical relationship a + b = c.

We also define a means of combination, so that primitive constraints can be combined to express more complex relations. In this way, our program resembles a programming language. We combine constraints by constructing a network in which constraints are joined by connectors. A connector is an object that "holds" a value and may participate in one or more constraints.

For example, we know that the relationship between Fahrenheit and Celsius temperatures is:

```
9 * c = 5 * (f - 32)
```

This equation is a complex constraint between c and f. Such a constraint can be thought of as a network consisting of primitive adder, multiplier, and constant constraints.




In this figure, we see on the left a multiplier box with three terminals, labeled a, b, and c. These connect the multiplier to the rest of the network as follows: The a terminal is linked to a connector celsius, which will hold the Celsius temperature. The b terminal is linked to a connector w, which is also linked to a constant box that holds 9. The c terminal, which the multiplier box constrains to be the product of a and b, is linked to the c terminal of another multiplier box, whose b is connected to a constant 5 and whose a is connected to one of the terms in the sum constraint.

Computation by such a network proceeds as follows: When a connector is given a value (by the user or by a constraint box to which it is linked), it awakens all of its associated constraints (except for the constraint that just awakened it) to inform them that it has a value. Each awakened constraint box then polls its connectors to see if there is enough information to determine a value for a connector. If so, the box sets that connector, which then awakens all of its associated constraints, and so on. For instance, in conversion between Celsius and Fahrenheit, `w`, `x`, and `y` are immediately set by the constant boxes to 9, 5, and 32, respectively. The connectors awaken the multipliers and the adder, which determine that there is not enough information to proceed. If the user (or some other part of the network) sets the `celsius` connector to a value (say 25), the leftmost multiplier will be awakened, and it will set `u` to `25 * 9 = 225`. Then `u` awakens the second multiplier, which sets `v` to 45, and `v` awakens the adder, which sets the `fahrenheit` connector to 77.

**Using the Constraint System.** To use the constraint system to carry out the temperature computation outlined above, we first create two named connectors, `celsius` and `fahrenheit`, by calling the `connector` constructor.

```
>>> celsius = connector('Celsius')
>>> fahrenheit = connector('Fahrenheit')
```

Then, we link these connectors into a network that mirrors the figure above. The function `converter` assembles the various connectors and constraints in the network.

```
>>> def converter(c, f):
        """Connect c to f with constraints to convert from Celsius to Fahrenheit."""
        u, v, w, x, y = [connector() for _ in range(5)]
        multiplier(c, w, u)
        multiplier(v, x, u)
        adder(v, y, f)
        constant(w, 9)
        constant(x, 5)
        constant(y, 32)

>>> converter(celsius, fahrenheit)
```

We will use a message passing system to coordinate constraints and connectors. Constraints are dictionaries that do not hold local states themselves. Their responses to messages are non-pure functions that change the connectors that they constrain.

Connectors are dictionaries that hold a current value and respond to messages that manipulate that value. Constraints will not change the value of connectors directly, but instead will do so by sending messages, so that the connector can notify other constraints in response to the change. In this way, a connector represents a number, but also encapsulates connector behavior.

One message we can send to a connector is to set its value. Here, we (the `'user'`) set the value of `celsius` to 25.

```
>>> celsius['set_val']('user', 25)
Celsius = 25
Fahrenheit = 77.0
```

Not only does the value of `celsius` change to 25, but its value propagates through the network, and so the value of `fahrenheit` is changed as well. These changes are printed because we named these two connectors when we constructed them.

Now we can try to set `fahrenheit` to a new value, say 212.

```
>>> fahrenheit['set_val']('user', 212)
Contradiction detected: 77.0 vs 212
```

The connector complains that it has sensed a contradiction: Its value is 77.0, and someone is trying to set it to 212. If we really want to reuse the network with new values, we can tell `celsius` to forget its old value:

```
>>> celsius['forget']('user')
Celsius is forgotten
Fahrenheit is forgotten
```

The connector `celsius` finds that the `user`, who set its value originally, is now retracting that value, so `celsius` agrees to lose its value, and it informs the rest of the network of this fact. This information eventually propagates to `fahrenheit`, which now finds that it has no reason for continuing to believe that its own value is 77. Thus, it also gives up its value.

Now that `fahrenheit` has no value, we are free to set it to 212:

```
>>> fahrenheit['set_val']('user', 212)
Fahrenheit = 212
Celsius = 100.0
```

This new value, when propagated through the network, forces `celsius` to have a value of 100. We have used the very same network to compute `celsius` given `fahrenheit` and to compute `fahrenheit` given `celsius`. This non-directionality of computation is the distinguishing feature of constraint-based systems.

**Implementing the Constraint System.** As we have seen, connectors are dictionaries that map message names to function and data values. We will implement connectors that respond to the following messages:

- `connector['set_val'](source, value)` indicates that the `source` is requesting the connector to set its current value to `value`.
- `connector['has_val']()` returns whether the connector already has a value.
- `connector['val']` is the current value of the connector.
- `connector['forget'](source)` tells the connector that the `source` is requesting it to forget its value.
- `connector['connect'](source)` tells the connector to participate in a new constraint, the `source`.

Constraints are also dictionaries, which receive information from connectors by means of two messages:

- `constraint['new_val']()` indicates that some connector that is connected to the constraint has a new value.
- `constraint['forget']()` indicates that some connector that is connected to the constraint has forgotten its value.

When constraints receive these messages, they propagate them appropriately to other connectors.

The `adder` function constructs an adder constraint over three connectors, where the first two must add to the third: `a + b = c`. To support multidirectional constraint propagation, the adder must also specify that it subtracts `a` from `c` to get `b` and likewise subtracts `b` from `c` to get `a`.

```
>>> from operator import add, sub
>>> def adder(a, b, c):
        """The constraint that a + b = c."""
        return make_ternary_constraint(a, b, c, add, sub, sub)
```

We would like to implement a generic ternary (three-way) constraint, which uses the three connectors and three functions from `adder` to create a constraint that accepts `new_val` and `forget` messages. The response to messages are local functions, which are placed in a dictionary called `constraint`.

```
>>> def make_ternary_constraint(a, b, c, ab, ca, cb):
        """The constraint that ab(a,b)=c and ca(c,a)=b and cb(c,b) = a."""
        def new_value():
            av, bv, cv = [connector['has_val']() for connector in (a, b, c)]
            if av and bv:
                c['set_val'](constraint, ab(a['val'], b['val']))
            elif av and cv:
                b['set_val'](constraint, ca(c['val'], a['val']))
            elif bv and cv:
                a['set_val'](constraint, cb(c['val'], b['val']))
        def forget_value():
            for connector in (a, b, c):
                connector['forget'](constraint)
        constraint = {'new_val': new_value, 'forget': forget_value}
        for connector in (a, b, c):
            connector['connect'](constraint)
        return constraint
```

The dictionary called `constraint` is a dispatch dictionary, but also the constraint object itself. It responds to the two messages that constraints receive, but is also passed as the `source` argument in calls to its connectors.

The constraint's local function `new_value` is called whenever the constraint is informed that one of its connectors has a value. This function first checks to see if both `a` and `b` have values. If so, it tells `c` to set its value to the return value of function `ab`, which is `add` in the case of an `adder`. The constraint passes *itself* (`constraint`) as the `source` argument of the connector, which is the adder object. If `a` and `b` do not both have values, then the constraint checks `a` and `c`, and so on.

If the constraint is informed that one of its connectors has forgotten its value, it requests that all of its connectors now forget their values. (Only those values that were set by this constraint are actually lost.)

A `multiplier` is very similar to an `adder`.

```
>>> from operator import mul, truediv
>>> def multiplier(a, b, c):
        """The constraint that a * b = c."""
        return make_ternary_constraint(a, b, c, mul, truediv, truediv)
```

A constant is a constraint as well, but one that is never sent any messages, because it involves only a single connector that it sets on construction.

```
>>> def constant(connector, value):
        """The constraint that connector = value."""
        constraint = {}
```

```
        connector['set_val'](constraint, value)
        return constraint
```

These three constraints are sufficient to implement our temperature conversion network.

**Representing connectors.** A connector is represented as a dictionary that contains a value, but also has response functions with local state. The connector must track the `informant` that gave it its current value, and a list of `constraints` in which it participates.

The constructor `connector` has local functions for setting and forgetting values, which are the responses to messages from constraints.

```
>>> def connector(name=None):
        """A connector between constraints."""
        informant = None
        constraints = []
        def set_value(source, value):
            nonlocal informant
            val = connector['val']
            if val is None:
                informant, connector['val'] = source, value
                if name is not None:
                    print(name, '=', value)
                inform_all_except(source, 'new_val', constraints)
            else:
                if val != value:
                    print('Contradiction detected:', val, 'vs', value)
        def forget_value(source):
            nonlocal informant
            if informant == source:
                informant, connector['val'] = None, None
                if name is not None:
                    print(name, 'is forgotten')
                inform_all_except(source, 'forget', constraints)
        connector = {'val': None,
                     'set_val': set_value,
                     'forget': forget_value,
                     'has_val': lambda: connector['val'] is not None,
                     'connect': lambda source: constraints.append(source)}
        return connector
```

A connector is again a dispatch dictionary for the five messages used by constraints to communicate with connectors. Four responses are functions, and the final response is the value itself.

The local function `set_value` is called when there is a request to set the connector's value. If the connector does not currently have a value, it will set its value and remember as `informant` the source constraint that requested the value to be set. Then the connector will notify all of its participating constraints except the constraint that requested the value to be set. This is accomplished using the following iterative function.

```
>>> def inform_all_except(source, message, constraints):
        """Inform all constraints of the message, except source."""
        for c in constraints:
            if c != source:
                c[message]()
```

If a connector is asked to forget its value, it calls the local function `forget-value`, which first checks to make sure that the request is coming from the same constraint that set the value originally. If so, the connector informs its associated constraints about the loss of the value.

The response to the message `has_val` indicates whether the connector has a value. The response to the message `connect` adds the source constraint to the list of constraints.

The constraint program we have designed introduces many ideas that will appear again in object-oriented programming. Constraints and connectors are both abstractions that are manipulated through messages. When the value of a connector is changed, it is changed via a message that not only changes the value, but validates it (checking the source) and propagates its effects (informing other constraints). In fact, we will use a similar architecture of dictionaries with string-valued keys and functional values to implement an object-oriented system later in this chapter.

*Continue*: 2.5 Object-Oriented Programming