39.3 Space/Time-Bounded Compression

As described in the previous chapter, it is impossible to write the "perfect" compression algorithm that requires the fewest bits to output some bitstream B.

Space-Bounded Compression

However, what about the problem of space-bounded compression? In this problem, we take in two inputs: a bitstream B and a target size S. The goal, then, is to find a program of length $\leq S$ that outputs B.

It turns out that such a problem is also uncomputable. If it were, then we could simply binary search on different values of S to find the optimal compression program size, which is impossible as shown in te previous section.

Space-Time-Bounded Compression

What if we take our problem from above, and add a constraint that we can run at most T lines of bytecode?

It might seem unintuitive, but this kind of problem is actually solvable. We will use the following algorithm:

```
for length L = 1....S:
   for each possible program P of length L:
     while (P is running && !(B is outputted) && lines_executed < T):
        run the next line of P</pre>
```

The runtime of this algorithm is $O(T * 2^S)$, and in the end, it will either output some program P that has the correct output and is bounded by T and S, or return that no such program is possible.

Efficient Bounded Compression

The runtime above is exponential in S. Thus, we might ask if it's possible to solve the space-time-bounded compression problem *efficiently*. As we'll see in the next chapter, this depends on our definition of efficiency.

Previous
39.2 Optimal Compression, Kolmogorov Complexity

Next
39.4 P = NP

Last updated 1 year ago

