

Lab 7

Left-Leaning Red Black Trees (LLRBs)



Announcements

Project 2A is due Wednesday, 3/06 at 11:59 pm.

Project 2B will be due Monday, 4/01 at 11:59 pm (this is the Monday after spring break!).



2-3 Trees (Balanced Search Trees)



Binary Search Trees

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We can't guarantee that binary search trees will always have a “bushy” structure. In the worst case, we would have a “spindly” tree.

- As a reminder, a bushy structure would have a worst case of $O(\log N)$ and a spindly tree would have a worst case runtime of $O(N)$ (and this applies for insertion and lookup).



Balanced Search Trees

Main Idea: Instead of adding a new node for each inserted element, we want to put **multiple** elements into a single node.

- Simply put, we always insert an element into an **existing node** and if the node becomes too big, we “split” the node.



Balanced Search Trees

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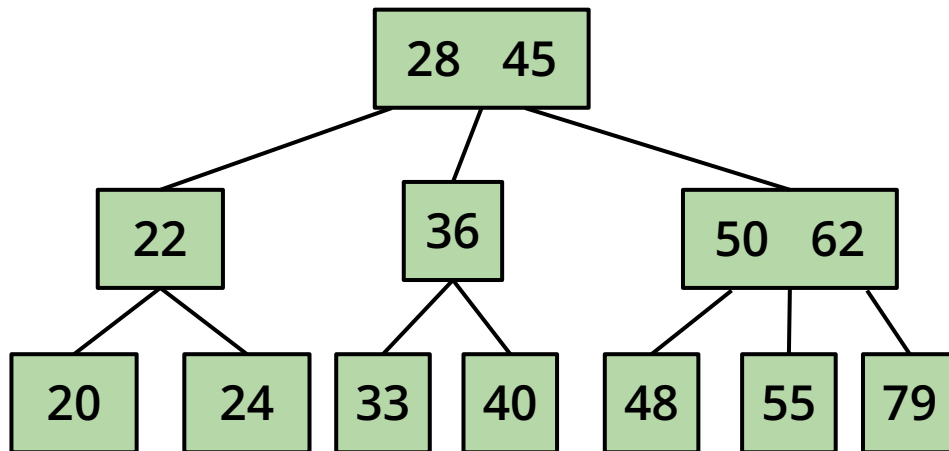
While there are several variants of a balanced search tree, we'll go with a 2-3 tree for this lab.

What this means is that:

- We can only have **2 items max per node**.
- And each node can have **3 children max**.



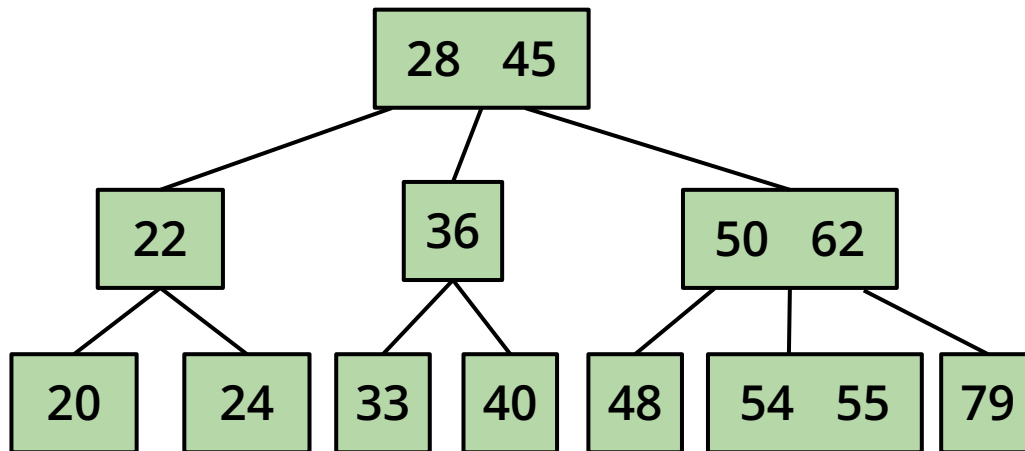
Balanced Search Trees



If we insert 54, where would that go?



Balanced Search Trees



It would go into the same node as 55!



Left-Leaning Red Black Trees

Now that we have balanced search trees down, let's introduce the **left-leaning red black trees**. In practice, balanced search trees are actually kind of hard to implement.



Left-Leaning Red Black Trees (LLRBs)



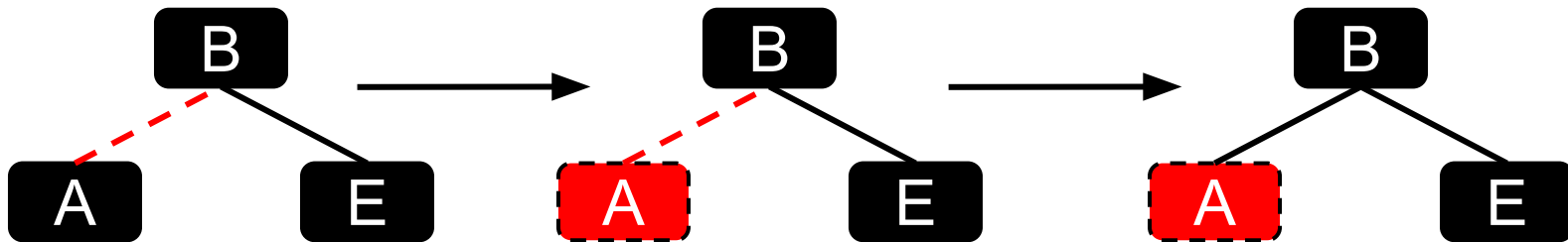
LLRBs

LLRBS are:

- A 2-3 tree represented in a binary search tree format (one item per node)
- This data structure is represented by two different types of nodes:
 - A **red** node means that the node is squished in the same node as the parent (in the 2-3 tree representation)
 - A **black** node means just a regular node in the 2-3 tree.



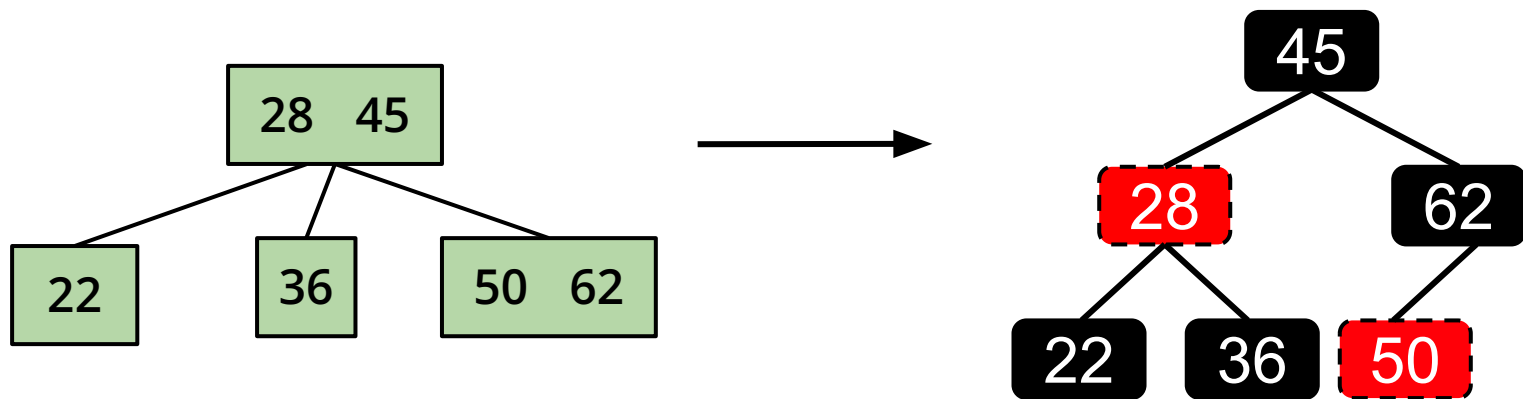
Links vs Nodes (Disclaimer)



Disclaimer: You may have wondered at this point why we're using colored nodes. In lecture, we introduce LLRBs with links. That is to say, if two nodes are connected with a red link, we say that the child node belongs in the same node as the parent in its 2-3 tree representation. For ease of implementation, the lab uses nodes instead (for the rest of lab, we'll use colored nodes).



LLRBs



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No **red** node can have a red parent (every **red** node's parent is black)

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- Or a red right child of a red node (which is effectively no right-leaning red node).



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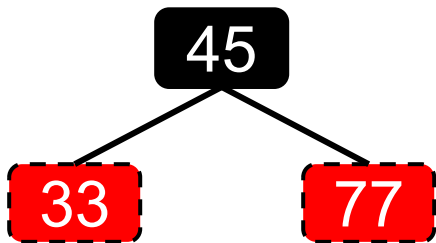
As an important note, these red nodes come from insertion - whenever we insert a node into an LLRB, it's always as a **red** node.



LLRB Operations



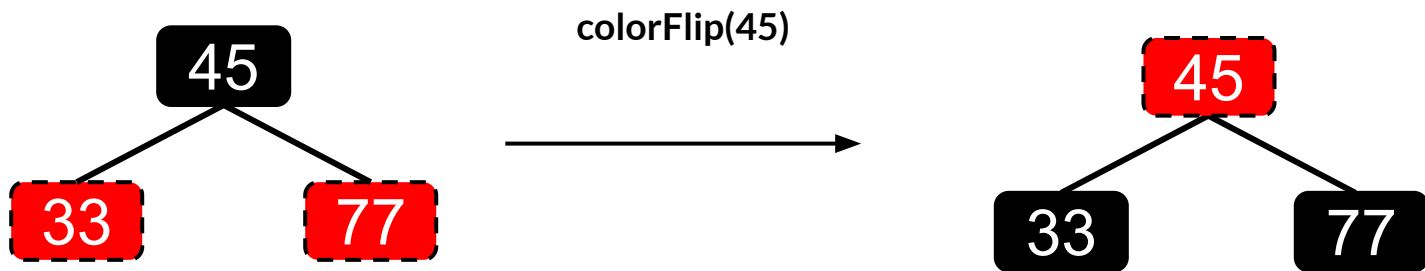
colorFlip



Which invariant is broken?



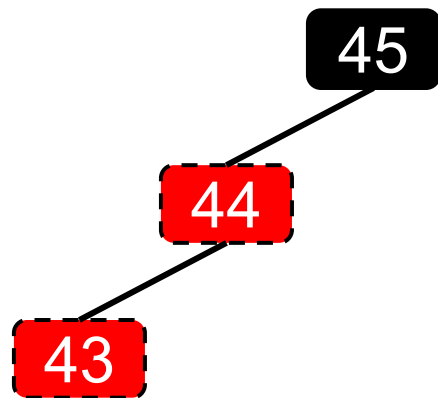
colorFlip



Invariant: No node can have two **red** children.



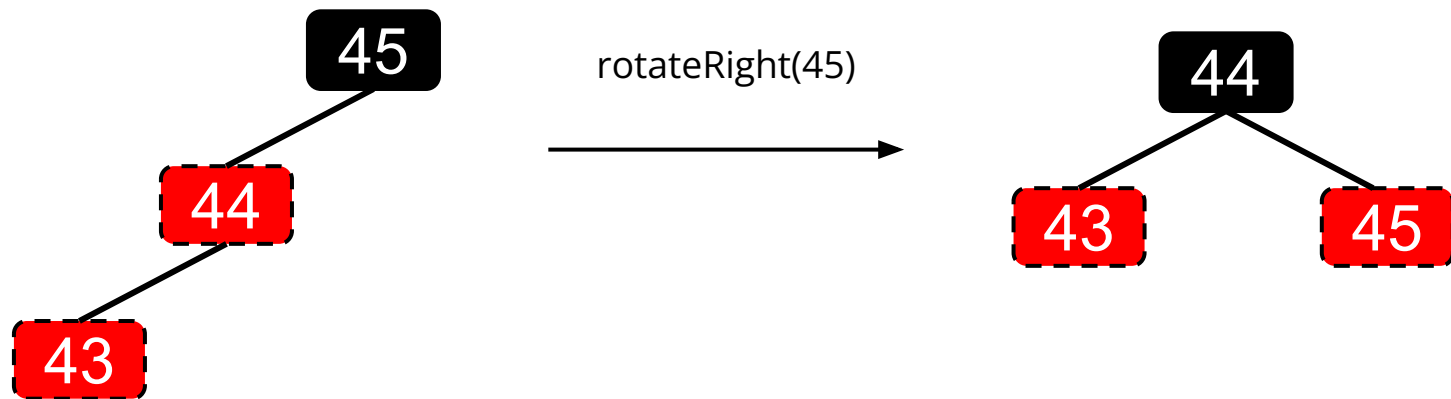
rotateRight



Which invariant is broken?



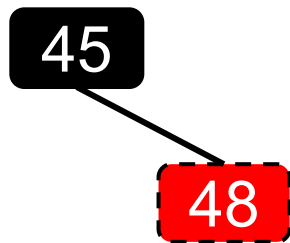
rotateRight



Invariant: The parent of a red node must be black (this means that we can not have two left-leaning red nodes).



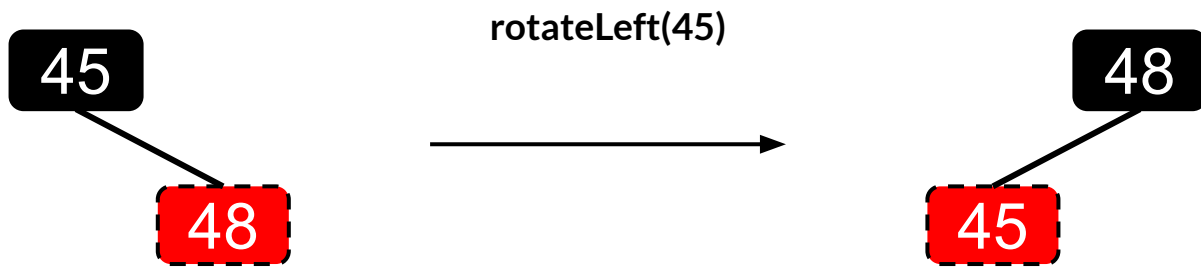
rotateLeft



Which invariant is broken?



rotateLeft



Invariant: All red nodes must be left-leaning.



Lab Overview



An Overview

Lab 08 is due Friday, 3/08 at 11:59 pm.

Deliverables:

- Complete the following methods in `RedBlackTree.java`
 - `flipColors`
 - `rotateRight` and `rotateLeft`
 - `insert`



Lab Notes

The lab provides a Node class for you to use (`static class RBTreeNode<T>`). Make sure to read through it first to understand what it encompasses before starting on the lab! It is in `RedBlackTree.java`.

The syntax to declare the node object is not the same as Proj1A, but should be like below:

```
RBTreeNode<T> node = ...;
```

