# Lecture 4A: Discrete Probability

UC Berkeley CS70
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# Announcements!

- Welcome to the **probability section**!
  - Lots of real-world applications & cool topics :)
  - Biggest advice: keep up with the material. Everything builds on past topics.
- For remote students, TAs will be answering questions on Ed now :)
  - Thanks to Casey & Harry! There will be an Ed thread for each lecture where you can ask questions.
- Read Weekly Post! (**HW threshold change**)

# You’ve all done probability before :)

If you flip a fair coin what’s the chance you get heads? What’s the chance you get an even number when you roll a fair dice?

At its essence, probability is just the chances of different ‘possibilities’ happening.

# Defining Probability Spaces

First, **define a probability space**:

1. **The sample space (Ω)** is the set of all possible outcomes ω

# Defining Probability Spaces (cont.)

$|\Omega|$ = the size of the sample space

# Defining Probability Space

You can combine sample points into **events** (which have complements!)**:**

# Defining Probability Spaces

**2. The probability of each sample point, P(ω),** where two things must be true:

1.
2.

# Defining Probability Spaces

We can also discuss **probabilities of events.**

# Uniform Probability Spaces

Let's look at **uniform probability spaces,** where every sample point has an equal probability of occurring**.**

# Uniform Probability Spaces (cont.)

If you roll two dice, what's the probability that your sum is 11?

# Non-Uniform Probability Space: Unfair Coin Tosses

What is the probability of getting HHTT with a coin that has probability P(heads) = ⅓? What about TTTT?

# Non-Uniform Probability Space: Unfair Coin Tosses

What is the chance of the **event** that all four of your coin tosses are the same?

# Birthday Paradox :)

Let’s say there are 75 people in this lecture hall. What’s the probability that two of us share a birthday? Make some guesses!

1. Less than 10%?
2. Between 10% and 50%?
3. Between 50% and 90%?
4. Greater than 90%?

# Birthday Paradox

In fact, we only need **23 people in a room** (ie your discussion sections!) to have the probability that two people share a birthday be > 50%.

# Balls in Bins

Let’s say I throw 10 balls into 5 bins, where each ball is **equally likely to land in any bin.** What’s the probability the first bin is empty?

# Balls in Bins (cont.)

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# Monty Hall Problem

There are three doors. Behind one is tickets to Taylor Swift's Eras tour (<3), but behind the other two are tickets to a John Mayer concert.

Should you make the switch?

# Monty Hall Problem (cont.)

Let’s look more into the math behind it :)

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# Recap

Whenever you're solving a probability problem, you **always** want to define the following:

1. What is the sample space? What is its size?
2. What is the probability of each sample point?
3. What is the event we're looking at? Is it easier to use its complement?
4. Finally, compute the probability of the event :)