Lecture 7A: Continuous Probability III

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Exponential Distribution

Def: For some $\lambda > 0$, a continuous random variable X is an <u>exponential random variable with parameter λ</u> if it has the following PDF,

$$f(x) = \begin{cases} \lambda e^{-\lambda x}, & \text{if } x \geq 0, \\ 0, & \text{otherwise,} \end{cases}$$

We can write $X \sim \text{Exp}(\lambda)$ if is an exponential random variable.

Let's check that f(x) satisfies the two PDF properties

Expectation & Variance of Exponential

For a random variable X ~ Exp(λ), $$\mathbb{E}[X] = \frac{1}{\lambda}$$

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For a random variable X ~ Exp(λ),

$$\mathrm{Var}(X) = \frac{1}{\lambda^2}.$$

Example Exponential

Going back to our terrible alarm clock, we know the behavior is:

Once plugged in, the alarm will randomly ring once after some amount of time, however we know it goes off at a rate of 1 time every 10 minutes.

Let X be the amount of time it takes the alarm to sound. $X \sim \text{Exp}(1/10)$, because our rate of rings is 1/10 per minute.

How many minutes should we expect to wait before the alarm rings?

An important property to note

$$\mathbb{P}[X > t] = e^{-\lambda t}$$

Proof:

Exponential Relation to Geometric

Let's try to draw a more rigorous connection between Exponential and Geometric distributions. Take our Geometric r.v. and consider running trials after every **δ** seconds. The probability of a success is p = λ**δ**.

Then, let Y denote the amount of time/seconds before a successful trial:

$$\mathbb{P}[Y > k\delta] = (1-p)^k = (1-\lambda\delta)^k,$$

To translate our trials to a continuum, consider taking the limit of d → 0. Then, for any time t,

$$\mathbb{P}[Y > t] = \mathbb{P}[Y > (\tfrac{t}{\delta})\delta] = (1-\lambda\delta)^{t/\delta} \approx e^{-\lambda t}$$

Continuous Example!

Let V and W have the joint density $12e^{-v-3w}$ for $0 < v < w < \infty$. Find the distribution of V.

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Another Continuous Example!

Let $f(x) = k(2x+3)$ for $-1 <= x <= 2$, and 0 otherwise. What is k?

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Min of Uniforms

We have n uniform(0, 2) random variables. For $0 < x < 2$, what is the probability that the minimum of the RVs is $\le x$?

Min of Exponentials

We have n exponential random variables with different parameters. Find the distribution of their minimum.

Transforming Exponentials

Let X be exponential(λ) and let $Y = 3X$. What distribution does Y follow?

Exponential

Let T have the exponential(λ) distribution and let N be the integer part of T. What is the distribution of N?

Exponential (cont.)

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Combining Exponential and Uniform

Let $X \sim \text{Exp}(\lambda)$ and let Y be Uniform $[0, X]$. What is the joint distribution of X and Y?

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Continuous Tail Sum (non-negative RV X)

$$\mathbf{E}(X) = \int_0^\infty [1 - F(x)]dx = \int_0^\infty \mathbf{P}(X > x)dx \ .$$

Continuous Tail Sum (non-negative RV X)

Memoryless Exponentials

For $X \sim Exp(\lambda)$ find $P(X > s + n \mid X > n)$

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For $X \sim Exp(\lambda)$ find $P(X > s + t \mid X > t)$

If you like probability :))

Recap

Practiced Continuous R.V.s:

- Finding marginals from joint densities
- Finding constants of integration, E(X), Var(X) from PDF

More Properties of Exponential/Uniform:

- Min of Exponentials/Uniforms
- Memoryless Property of Exponential
- Continuous Tail Sum