UC Berkeley
Department of Electrical Engineering and Computer Sciences

EE126: Probability and Random Processes

Discussion 7

Date: Wednesday, March 9, 2016

Problem 1. *Splitting and Merging a Poisson Process*

a. Packets arrive at a node of a data network according to a Poisson process of rate λ. Each packet is a local packet with probability p or a transit packet with probability $1-p$. Show that the process of local packet arrivals is a Poisson process with rate λp.

b. People with letters to mail arrive at the post office according to a Poisson process with rate λ_1, while people with packages to mail arrive according to an independent Poisson process with rate λ_2. Show that the merged process, which includes arrivals of both types is Poisson with rate $\lambda_1 + \lambda_2$.

Problem 2. *More Splitting and Merging*

Consider two independent Poisson processes with rates λ_1 and λ_2. Those processes measure the number of customers arriving in store 1 and 2.

(a) What is the probability that a customer arrives in store 1 before any arrives in store 2?

(b) What is the probability that in the first hour exactly 6 customers arrive at the two stores? (The total for both is 6)

(c) Given exactly 6 have arrived at the two stores, what is the probability all 6 went to store 1?

Problem 3. *Markov Chains Meet Linear Algebra*

Consider the transition matrix:

$$P = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 0 & 1/2 & 1/2 \\ 0 & 0 & 1 \end{bmatrix}$$

(a) Find P^n.

Hint: This can be done without any math.

(b) Find the distinct eigenvalues of P along with their multiplicities.

(c) Can you write $P = U\Lambda U^{-1}$ for some diagonal matrix Λ and invertible matrix U?

Problem 4. The Forgotten Process

Let $Y = X_1 + X_2 + X_3 + \cdots + X_N$ where the random variables X_i are geometric with parameter p, and N is geometric with parameter q. Assume that the random variables $N, X_1, \ldots X_N$ are independent. Show that Y is geometric with parameter pq.