UC Berkeley
Department of Electrical Engineering and Computer Sciences

EE126: Probability and Random Processes

Discussion 8
Spring 2016

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Problem 1 (FA06 Final P4). At the nuclear power plant, one of the reactors begins to melt down. The emission of radioactive particles can be modeled as a Poisson process with rate $\lambda = 100$ particles/second.

(a) What is the expected total number of particles that escape the time window $[10, 20] \cup [50, 60]$ seconds?

(b) Suppose that exactly 300 particles escaped in the first second. What is the PDF of the number of particles that escape in the next second?

In order to stop the spread of radiation, the plant is equipped with a set of n shields that are either ON or OFF. Any OFF shield has no effect on the particle stream, whereas any ON shield blocks each particle with probability p, and let it through with probability $1 - p$, independently of all other particles. Suppose that each shield acts independently of all the other shields.

(c) For some fixed integer k (with $1 \leq k \leq n$), suppose that exactly k of the shields are ON, and consider the stochastic process defined by the particles that end up escaping. Prove that the expected number of particles that escape per second is equal to $100(1 - p)^k$. Is this a Poisson process?

(d) Now suppose that each shield is turned ON (independently of all other shields) with probability 0.50. Let X_n represent the number of particles that escape in the first second. (Recall that fixed integer n is the total number of shields.) What is $E[X_n]$? What is $\text{var}(X_n)$?

(e) Does the sequence $\{X_n\}$ converge in probability?

Problem 2. A machine, once in production mode, operates continuously until an alarm signal is generated. The time up to the alarm signal is an exponential random variable with parameter 1. Subsequent to the alarm signal, the machine is tested for an exponentially distributed amount of time with parameter 5. The test results are positive, with probability 1/2, in which case the machine returns to production mode, or negative, with probability 1/2, in which case the machine is taken for repair. The duration of the repair is exponentially distributed with parameter 3.

(a) Let states $1, 2, 3$ correspond to production mode, testing, and repair, respectively. Let $X(t), t \geq 0$ denote the state of the system at time t. Is $X(t)$ a CTMC?

(b) Find the rate and transition matrices.

(c) Find the steady state probabilities.

Problem 3. A two-dimensional Poisson process is a process of randomly occurring special points in the plane such that (i) for any region of area A the number of special points in that region has a Poisson distribution with mean λA, and (ii) the number of special points in non-overlapping regions is independent. For such a process consider an arbitrary location in the plane and let X denote its distance from its nearest special point (where distance between two points (x_1, y_1) and (x_2, y_2) is defined as $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$). Show that

(a) $\Pr(X > t) = \exp(-\lambda \pi t^2)$.

(b) $\mathrm{E}[X] = \frac{1}{2\sqrt{\lambda}}$.