UC Berkeley
Department of Electrical Engineering and Computer Sciences

EE126: Probability and Random Processes

Problem Set 1
Spring 2016

Issued: Tuesday, January 19, 2016 **Due:** 8am, Thursday, January 28, 2016

Problem 1. Find an example of 3 events A, B, and C such that each pair of them are independent, but they are not mutually independent. Show the calculations.

Problem 2. There are two coins in front of you, one is fair, and the other is biased such that the probability of heads is $\frac{3}{4}$. You pick one of the two at random and flip it 4 times. The coin comes up heads twice. What is the probability that the coin is fair?

Problem 3. Jim and George are setting up venture capital portfolios. Suppose that Jim picks $n+1$ startups to fund and George picks n startups to fund. Suppose that the probability of any startup succeeding is $\frac{1}{2}$ and all of the startups succeed or fail independently. What is the probability that Jim picks more successful startups than George?

Problem 4. Alice and Bob are playing a game where an unfair coin with probability p of heads is tossed n times. Alice wins the game if the coin comes up heads an even amount of times and Bob wins the game if the coin comes up heads an odd number of times. What is the probability that Alice wins? To get some inuition, let p be for example $\frac{2}{3}$, what happens as $n \to \infty$?

Problem 5. The NBA is looking to expand to another city. In order to decide which city will receive a new team, the commissioner interviews potential owners from each of the N potential cities one at a time. Unfortunately, the owners would like to know immediately after the interview whether their city will receive the team or not. The commissioner decides to use the following strategy: she will interview the first m owners and reject all of them. After the mth owner is interviewed, she will pick the first city that is better than all previous cities. What is the probability that the best city is selected? Assume that the commissioner has an objective method of scoring each city and that each city receives a unique score.

Problem 6. Consider two strange countries, A and B. There are n cities with airports in country A and m cities with airports in country B. Let us call these cities $A_1, A_2, \ldots, A_n$ and $B_1, B_2, \ldots, B_m$. The airports are such that no domestic flights are possible, i.e. there are no flights between (A_i, A_j) and (B_i, B_j). For each pair of cities in different groups, i.e., (A_i, B_j), there is a flight between these two cities with probability p, independently from all other pairs. An example of the flight connection $(n = 4, m = 4)$ is shown in Figure 1(a). Now, suppose a person lives in city A_1, and let $N_2(A_1)$ be the set of cities that this person can reach by taking *at most* 2 flights. We call $N_2(A_1)$ the two-flight neighborhood of A_1. An example of two-flight neighborhood is shown in Figure 1(b). What is the probability that there is at least one city other than A_1 in $N_2(A_1)$, and at the same time, for every city in $N_2(A_1)$ other than A_1 itself, there is a *unique* flight route with at most 2 flights from A_1 to that city?

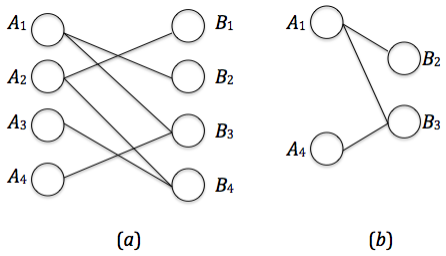

Figure 1: flight connection and two-flight neighborhood of A_1