UC Berkeley
Department of Electrical Engineering and Computer Sciences

EE126: Probability and Random Processes

Problem Set 7
Spring 2016

Issued: Thursday, March 10, 2016 **Due:** 11:59PM, Thursday, March 17, 2016

Problem 1. Michael misses shots with probability $\frac{1}{4}$, independent of other shots.

(a) What is the expected number of shots that Michael will make before he misses three times?

(b) What is the probability that the second and third time Michael makes a shot will occur when he takes his eigth and ninth shots, respectively?

(c) What is the probability that Michael misses two shots in a row before he makes two shots in a row?

Problem 2. Starting at time 0, the F line makes stops at Cory Hall according to a Poisson process of rate λ. Students arrive at the stop according to an independent Poisson process of rate μ. Every time the bus arrives, all students waiting get on.

(a) Given that the interarrival time between bus $i-1$ and bus i is x, find the distribution for the number of students entering the ith bus.

(b) Given that a bus arrived at 9:30AM, find the distribution for the number of students that will get on the next bus.

(c) Find the distribution of the number of students getting on the next bus to arrive after 11:00AM. (You can assume that time 0 was infinitely far in the past)

Problem 3. Consider a Poisson process $\{N_t, t \geq 0\}$ with rate $\lambda = 1$. Let random variable S_i denote the time of the i-th arrival.

(a) Given $S_3 = s$, find the joint distribution of S_1 and S_2.

(b) Find $\mathbb{E}[S_2|S_3 = s]$.

(c) Find $\mathbb{E}[S_3|N_1 = 2]$.

Problem 4. Each morning, as you pull out of your driveway, you would like to make a U-turn rather than drive around the block. Unfortunately, U-turns are illegal and police cars drive by according to a Poisson process with rate λ. You decide to make a U-turn one you see that the road has been clear of police cars for τ units of time. Let N be the number of police cars you see before you make a U-turn.

(a) Find $\mathbb{E}[N]$.

(b) Find the conditional expectation of the time elapsed between police cars $n-1$ and n, given that $N \geq n$.

(c) Find the expected time that you wait until you make a U-turn.

Problem 5. Team A and Team B are playing an untimed basketball game in which the two teams score points according to independent Poisson processes. Team A scores points according to a Poisson process with rate λ_A and Team B scores points according to a Poisson process with rate λ_B. The game is over when one of the teams has scored k more points than the other team. Find the probability that team A wins.