#1

EECS140 Solution Fall '09
HW4, due Thursday 9/24 at 8am
(This was Midterm 1, Spring 2008)

Name________________________

SID________________________

Prob.	Score
1	/10
2	/30
3	/20
4	/15
5	/25
Total	

1) Fill in the following table. Do NOT use a calculator!

dB	Power ratio	
13	$10 \times 2 = 20$	($\because$ 13dB = 10dB + 3dB)
-7	$\frac{1}{10} \times 2 = 0.2$	($\because$ −7dB = −10dB + 3dB)
16	$10 \times 2 \times 2 = 40$	($\because$ 16dB = 10dB + 3dB + 3dB)
-1	$0.2 \times 2 \times 2 = 0.8$	($\because$ −1dB = −7dB + 3dB + 3dB)
-9	$0.5 \times 0.5 \times 0.5 = 0.125$	($\because$ −9dB = −3dB − 6dB)
1	$10/8 = 1.25$	($\because$ 1dB = 10dB − 9dB)
17	$100/2 = 50$	($\because$ 17dB = 20dB − 3dB)
4	$10/4 = 2.5$	($\because$ 4dB = 10dB − 6dB)
-14	$\frac{1}{100} \times 2 \times 2 = 0.04$	($\because$ −14dB = −20dB + 6dB)

(If you include this factor, you would have got slightly smaller L & W, which is also acceptable.)

2) Design an NMOS-input common source amplifier with a PMOS load with a low frequency gain of approximately 200, a unity gain frequency of 1G rad/sec with a 1pF load, and an output swing of at least 500mV to 2V with a 2.5V single-sided supply. Clearly indicate what values you are using for g_m, r_o, I_D, V_{dsat}, gate bias, W, L for each transistor. Assume our standard process model. $(1+\lambda V_{DS}) \simeq 1$ is assumed

	g_m	r_o	I_D	V_{dsat}	V_G	W	L
NMOS	1mS	400kΩ	250μA	0.5V	1V	100μm	10μm
			100μA	0.2V	0.7V	100μm	4μm
			50μA	0.1V	0.6V	100μm	2μm
PMOS	1mS	400kΩ	250μA	0.5V	1.5V	200μm	10μm
			100μA	0.2V	1.8V	200μm	4μm
			50μA	0.1V	1.9V	200μm	2μm

There are countless number of ways to meet the specification. You'll get the full point as long as your design satisfies the following equations.

spec.

$A_V = 200$

$C_L = 1pF$

$w_u = 1G rad/s$

$V_{DSAT} \le 500mV$

$$- \; w_u = A_V \cdot w_p = g_m R_o \cdot \frac{1}{R_o \cdot C_L}$$

$$\Rightarrow g_m = 1mS$$

$$\Rightarrow R_{out} = \frac{A_V}{g_m} = 200k.$$

$$r_{on} \stackrel{?}{=} r_{op} = 400k$$ (You can choose other values as long as $(r_{on} \| r_{op}) = 200k$)

Now, you need to pick either V_{DSAT} or I_D. Let's fix V_{DSAT} first,

$$I_D = \frac{g_m \cdot V_{DSAT}}{2}, \quad V_{G,n} = V_{DSAT} + V_t, \quad V_{G,p} = 2.5 - (V_{DSAT} + V_t)$$

$$\lambda = \frac{1}{r_o I_D} = \frac{1}{L(\mu m)}, \quad \therefore L = r_o I_D$$

$$\left(\frac{W}{L}\right) = \frac{g_m}{\mu_n C_{ox} \cdot V_{DSAT}} \quad \therefore W = \frac{g_m \cdot L}{\mu_n C_{ox} V_{DSAT}}$$

Some example design values are given in the table.

3) Fill in the following table where each row is a different single-pole amplifier

G_m [S]	R_o [Ω]	C_L [F]	Av	ω_p [rad/s]	ω_u [rad/s]
50u	1M	50f	$50u \cdot 1M = 50$	$\frac{1}{1M \times 50f} = 20M$	$50 \cdot 20M = 1G$
$200/250K = 0.8m$	$\frac{1}{40M \times 100f} = 250K$	100f	200	40M	$200 \cdot 40M = 8G$
30m	$150/30m = 5K$	$30m/15G = 2p$	150	$15G/150 = 100M$	15G
$500/1M = 0.5m$	1M	$\frac{1}{1M \times 50M} = 20f$	$25G/50M = 500$	50M	25G

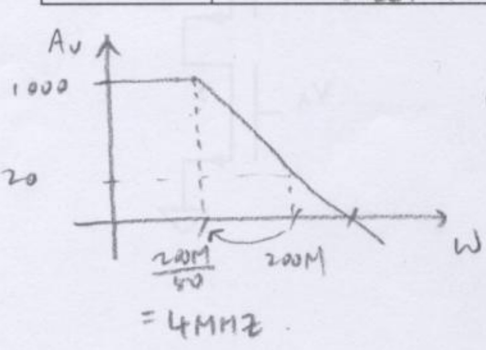
$$A_v = G_m \cdot R_o$$

$$\omega_p = \frac{1}{R_o \cdot C_L}$$

$$\omega_u = A_v \cdot \omega_p = \frac{g_m}{C_L}$$

4) You have a single-pole amplifier with a gain of 20 at 200MHz, and a low frequency gain of 1000. What is the gain at 100kHz, 10MHz, and 2GHz?

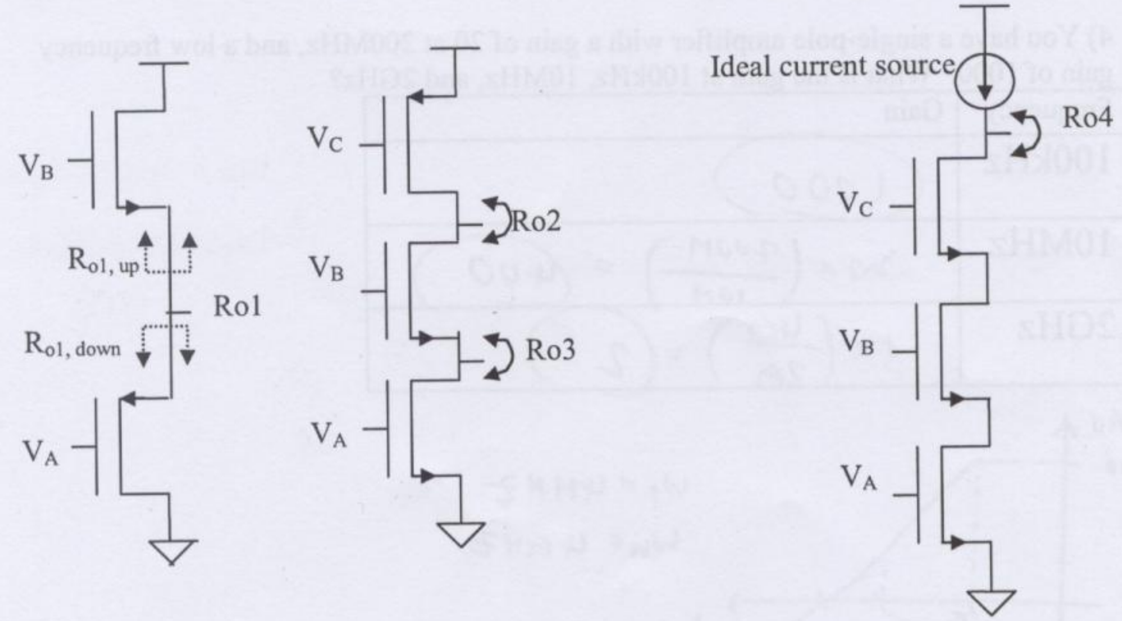
Frequency	Gain
100kHz	1000
10MHz	$20 \times \left(\frac{200M}{10M}\right) = 400$
2GHz	$1 \times \left(\frac{4G}{2G}\right) = 2$



ω_p = 4MHZ
ω_u = 4GHZ

5) What is the total low frequency impedance and the low frequency impedance seen "looking up" and "looking down" at the output node indicated in each circuit? Write your answer in terms of g_{mp} , g_{mn}, r_{on}, and r_{op}. Assume that all nmos devices have transconductance g_{mn} and output resistance r_{on}, and all pmos devices have transconductance g_{mp} and output resistance r_{op}. Write the full expression for up and down, and then the simplified total impedance assuming that gm*ro >> 1 for all combinations of gm and ro. You may ignore all capacitors.

	Full expression	Simplified expression for Ro, assuming gm ro >>1
$R_{o1,\,up}$	$\frac{1}{g_{mn}} \parallel r_{on}$	$\frac{1}{g_{mn}} \parallel \frac{1}{g_{mp}} = \frac{1}{g_{mn}+g_{mp}}$
$R_{o1,\,down}$	$\frac{1}{g_{mp}} \parallel r_{op}$	
$R_{o2,up}$	r_{op}	r_{op}
$R_{o2,down}$	$\overset{B}{r_{on}} + \overset{A}{r_{on}} + g_{mn} \cdot r_{on}^2$	
$R_{o3,up}$	$\frac{r_{on}+r_{op}}{1+g_{mn} r_{on}} = \frac{1}{g_{mn}}\left(\frac{r_{on}+r_{op}}{\frac{1}{g_{mn}}+r_{on}}\right)$	$r_{on} \parallel \frac{1}{g_{mn}}\left(\frac{1+\frac{r_{op}}{r_{on}}}{1+\frac{1}{g_{mn} r_{on}}}\right) \approx r_{on} \parallel \frac{1}{g_{mn}}\left(1+\frac{r_{op}}{r_{on}}\right)$
$R_{o3,down}$	r_{on}	
$R_{o4,up}$	∞	$g_{mn} \cdot r_{on} \cdot g_{mn} \cdot r_{on}^2 = g_{mn}^2 r_{on}^3$
$R_{o4,down}$	$r_{on} + R_{o2,down} + g_{mn} \cdot r_{on} \cdot R_{o2,down}$	



[prob. 6]

(1) Node A: $G_m = g_{mn}$

$$R_{o3} \simeq r_{on} \,||\, \frac{1}{g_{mn}}\left(1+\frac{r_{op}}{r_{on}}\right)$$

$$A_{v,A\to o3} = -G_m R_{o3}$$

$$\therefore \underline{C_{in} = C_{gs} + C_{gd}\left[1 + g_{mn}\left(r_{on} \,||\, \frac{1}{g_{mn}}\left(1+\frac{r_{op}}{r_{on}}\right)\right)\right]}$$

If we assume that $r_{on} \simeq r_{op}$, $r_{on} \gg \frac{1}{g_{mn}}$,

$$C_{in} \simeq C_{gs} + C_{gd}\left(1 + g_{mn}\cdot\frac{2}{g_{mn}}\right)$$

$$= C_{gs} + 3\,C_{gd}$$

(2) Node B:

see appendix for derivation

$$G_{m,B\to o3} = \frac{g_{mn} r_{on}}{r_{on}+r_{op}} \qquad G_{m,B\to o2} = \frac{-g_{mn}}{2+g_{mn} r_{on}}$$

$$R_{o,B\to o3} = \frac{r_{on}+r_{op}}{2+g_{mn}r_{on}+\frac{r_{op}}{r_{on}}}, \qquad R_{o,B\to o2} \simeq r_{op},$$

$$A_{v,B\to o3} = \frac{g_{mn}r_{on}}{2+g_{mn}r_{on}+\frac{r_{op}}{r_{on}}} \qquad A_{v,B\to o2} = \frac{-g_{mn}r_{op}}{2+g_{mn}r_{on}}$$

$$\therefore \underline{\underline{C_{in}}} = (1-A_{v,B\to o3})\,C_{gs} + (1-A_{v,B\to o2})\cdot C_{gd}$$

$$= \left(\frac{2+\frac{r_{op}}{r_{on}}}{2+g_{mn}r_{on}+\frac{r_{op}}{r_{on}}}\right)C_{gs} + \left(\frac{2+g_{mn}(r_{on}+r_{op})}{2+g_{mn}r_{on}}\right)C_{gd}$$

If we assume $r_{on} \simeq r_{op}$, $g_{mn}r_{on} \gg 1$.

$$C_{in} \simeq 2\cdot C_{gd}$$

(3) Node C: $G_m = g_{mp}$ $\qquad A_{v,C\to o2} = -g_{mp}\,r_{op}$

$$R_{o2} \simeq r_{op}.$$

$$\therefore \underline{C_{in} = C_{gs} + (1+g_{mp}r_{op})\,C_{gd}}$$

[prob. 7] Above ω_u

(1) Node A : $A_{v,A\to 03} \to 0$

$\therefore$ $C_{in} \simeq C_{gs} + C_{gd}$.

(2) Node B : $A_{v,B\to 02}$, $A_{v,B\to 03} \to 0$

$\therefore$ $C_{in} \simeq C_{gs} + C_{gd}$

(3) Node C : $A_{v,C\to 02} \to 0$

$\therefore$ $C_{in} \simeq C_{gs} + C_{gd}$

[prob. 8]

$A_v = -27$ (mid-range gain)

$C_{gs} = \frac{2}{3} W L C_{ox} + W \cdot C_{ol}'$

$= \frac{2}{3} \times 100\mu m \times 1\mu m \times 5fF/\mu m^2 + 100\mu m \cdot 0.5fF/\mu m$

$= 333.33fF + 50.0fF = \underline{383.3fF}$.

$C_{gd} = W \cdot C_{ol}' = \underline{50.0fF}$

(1) low frequency approximation.

$C_{in} = C_{gs} + (1 - A_v) C_{gd}$

$= 383.3fF + 28 \times 50.0fF$

$= 383.3fF + 1400.0fF$

$= \underline{1783.3fF}$ (overlap capacitance of C_{gd} dominates the overall cap.)

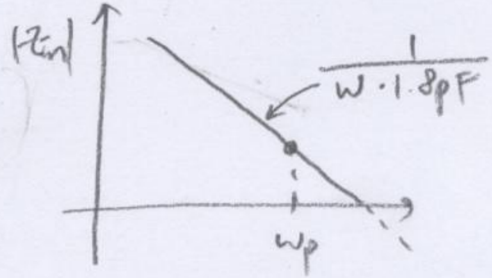
(2) high freq. approx

$C_{in} \simeq C_{gs} + C_{gd}$

$= \underline{433.3fF}$

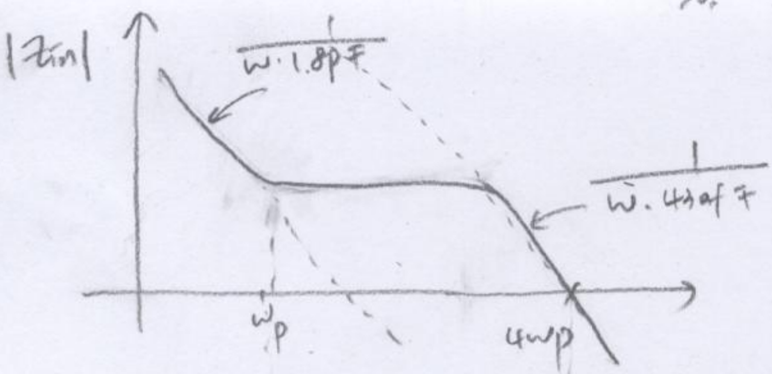
[prob. 9]

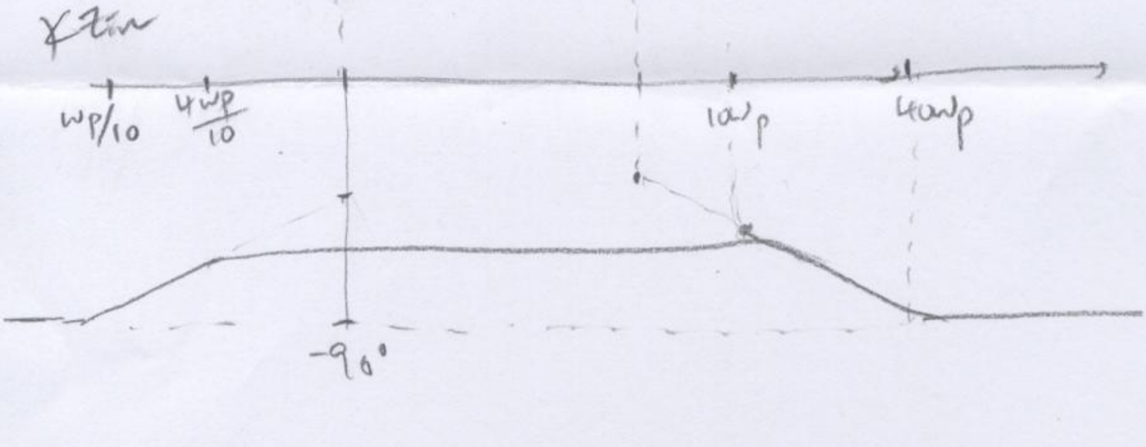
- low frequency $C_{in} \simeq 1.8pF$



- high freq. $C_{in} \simeq 430fF$. ×4 lower C

so, ×4 higher |Z|

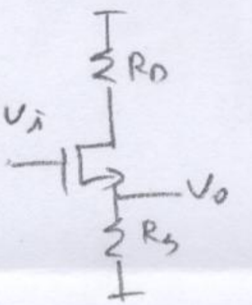




Appendix

< prob6. Node B gain derivation >

(1) $\underline{B \rightarrow O3}$



$$G_m = \frac{g_m}{1+\frac{R_D}{r_o}} = \frac{g_m r_o}{r_o + R_D}$$

$$R_O = \frac{r_o + R_D}{1+g_m r_o} \parallel R_S = \frac{R_S(r_o+R_D)}{R_S(1+g_m r_o) + r_o + R_D}$$

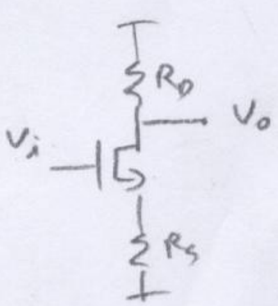
$$\boxed{A_v = \frac{g_m R_S}{1 + g_m R_S + \frac{R_S + R_D}{r_o}}}$$

* If, $\frac{R_S + R_D}{r_o} \ll 1$, $\quad A_v \simeq \underline{\frac{g_m R_S}{1 + g_m R_S}}$.

* In prob6, $R_S = r_{on}$, $R_D = r_{op}$, $g_m = g_{mn}$, $r_o = r_{on}$

$$\therefore A_v = \underline{\frac{g_m r_{on}}{2 + g_m r_{on} + \frac{r_{op}}{r_{on}}}}$$

(2) $B \rightarrow O2$.



$$G_m = \frac{-g_m}{1 + g_m R_S + \frac{R_S}{r_o}}$$

$$R_O = (r_o + R_S + g_m r_o R_S) \parallel R_D \simeq R_D$$

$$\boxed{A_v = \frac{-g_m R_D}{1 + g_m R_S + \frac{R_S}{r_o}}}$$

* If. $\frac{R_S}{r_o} \ll 1$ $\quad A_v \simeq \underline{\frac{g_m R_D}{1 + g_m R_S}}$

* In prob6, $R_S = r_{on}$, $R_D = r_{op}$, $g_m = g_{mn}$, $r_o = r_{on}$.

$$\therefore A_v = \underline{\frac{-g_{mn} r_{op}}{2 + g_{mn} r_{on}}}$$