EE C128 / ME C134 – Feedback Control Systems

Lecture – Chapter 11 – Design via Frequency Response

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Lecture abstract

Topics covered in this presentation

- FR compensator design advantages
- Lag, lead, & lag-lead compensators

Chapter outline

- 11 Design via frequency response
  - 11.1 Introduction
  - 11.2 Transient response via gain adjustment
  - 11.3 Lag compensation
  - 11.4 Lead compensation
  - 11.5 Lag-lead compensation

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RL vs. FR design, [1, p. 626]

RL

- Stability & TR design via gain adjustment
  - Repeated trials
- TR design via cascade compensation
  - Intuitive
- Steady-state error design via cascade compensation
  - Repeated trials

FR

- Stability & TR design via gain adjustment
  - Read gain from the plots
- TR design via cascade compensation
  - Repeated trials
- Steady-state error design via cascade compensation
  - Design derivative compensation & steady-state error jointly

FR design review, [1, p. 627]

Stability

- Nyquist criterion $\rightarrow$ stability
  - CL stable if OL stable & OL magnitude FR has a gain less than $0\ dB$ at the frequency where the phase FR is $180°$

TR

- $\downarrow \%OS \propto \uparrow \Phi_M$
- $\uparrow$ speed of response $\propto \uparrow$ bandwidth

Steady-state error

- $\downarrow$ steady-state error $\propto \uparrow$ low-frequency magnitude responses

- 11 Design via frequency response
  - 11.1 Introduction
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Φ_M-TR-gain relation, [1, p. 627]

Concept

- ζ (& $\%OS$) relate to Φ_M

OL TF

$$G(s) = \frac{\omega_n^2}{s(s + 2\zeta\omega_n)}$$

CL TF

$$T(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

ζ-Φ_M relation

$$\Phi_M = \tan^{-1}\left(\frac{2\zeta}{\sqrt{-2\zeta^2+\sqrt{1+4\zeta^4}}}\right)$$

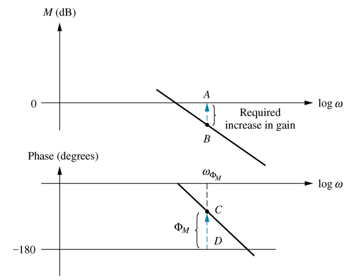

Figure: Bode plots showing gain adjustment for a desired Φ_M

Φ_M-TR-gain relation, [1, p. 627]

Procedure

1. Draw the Bode magnitude & phase plots for a convenient value of gain
2. Determine the required Φ_M from the $\%OS$

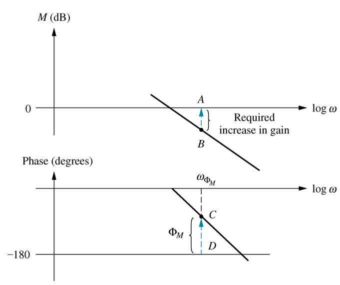

Figure: Bode plots showing gain adjustment for a desired Φ_M

Φ_M-TR-gain relation, [1, p. 628]

Procedure

3. Find the frequency, ω_{Φ_M}, on the Bode phase diagram that yields the desired Φ_M
4. Change the gain by an amount to force the magnitude curve to go through $0\ dB$ at ω_{Φ_M}. The amount of gain adjustment is the additional gain needed to produce the required Φ_M

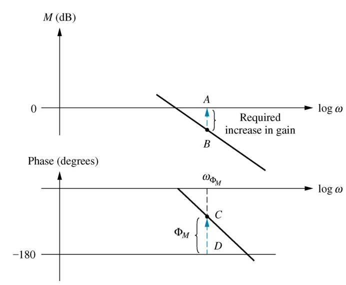

Figure: Bode plots showing gain adjustment for a desired Φ_M

Example, [1, p. 628]

Example (TR design via gain adjustment)

- *Problem:* For the position control system, find the value of the preamplifier gain, K, to yield $\%OS = 9.5\%$ in the TR for a step input
- *Solution:* On the board

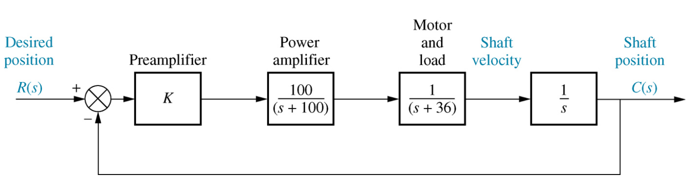

Figure: System

- 11 Design via frequency response
  - 11.1 Introduction
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Visualizing lag compensation, [1, p. 630]

Concept

- Improve the static error constant by ↑ only the low-frequency gain without any resulting instability
- ↑ Φ_M of the system to yield the desired TR

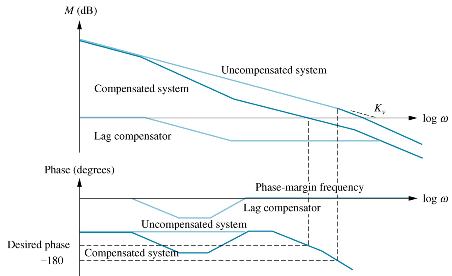

Figure: Visualizing lag compensation

Visualizing lag compensation, [1, p. 630]

Procedure

1. Set the gain, K, to the value that satisfies the steady-state error specification and plot the Bode magnitude and phase diagrams for this value of gain

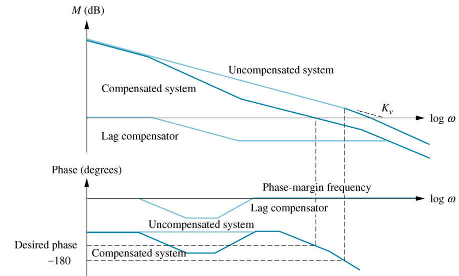

Figure: Visualizing lag compensation

Visualizing lag compensation, [1, p. 630]

Procedure

2. Find the frequency where Φ_M is $5°$ to $12°$ greater than the Φ_M that yields the desired TR. This compensates for the fact that the phase of the lag compensator may still contribute anywhere from $-5°$ to $-12°$ of phase at ω_{Φ_M}

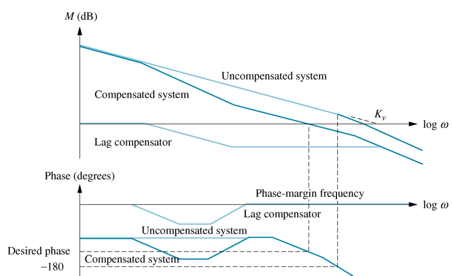

Figure: Visualizing lag compensation

Visualizing lag compensation, [1, p. 630]

Procedure

3. Select a lag compensator whose magnitude response yields a composite Bode magnitude diagram that goes through $0\ dB$ at the frequency found in Step 2

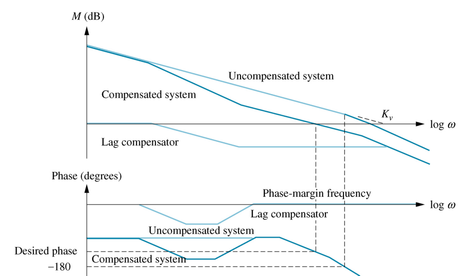

Figure: Visualizing lag compensation

Visualizing lag compensation, [1, p. 630]

Procedure

3.1 Draw the compensator's high-frequency asymptote to yield $0\ dB$ for the compensated system at the frequency found in Step 2

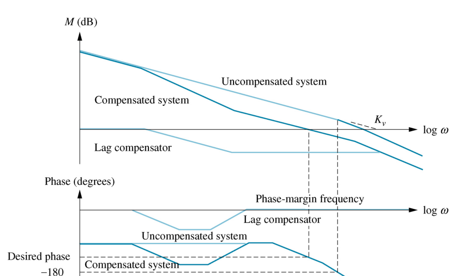

Figure: Visualizing lag compensation

Visualizing lag compensation, [1, p. 630]

Procedure

3.2 Select the upper break frequency to be 1 decade below the frequency found in Step 2

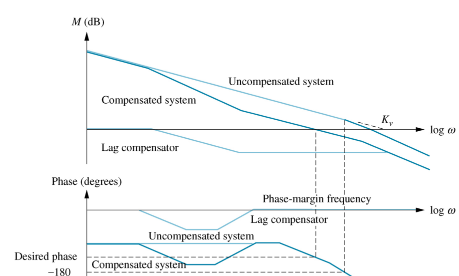

Figure: Visualizing lag compensation

Visualizing lag compensation, [1, p. 630]

Procedure

3.3 Select the low-frequency asymptote to be at 0 dB

3.4 Connect the compensator's high- & low-frequency asymptotes with a -20 dB/decade line to locate the lower break frequency

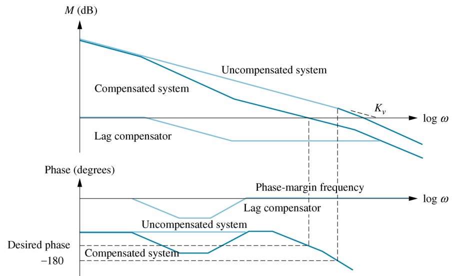

Figure: Visualizing lag compensation

Visualizing lag compensation, [1, p. 630]

Procedure

4. Reset the system gain, K, to compensate for any attenuation in the lag network in order to keep the static error constant the same as that found in Step 1

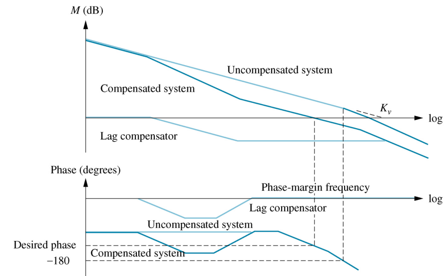

Figure: Visualizing lag compensation

Visualizing lag compensation, [1, p. 630]

Result

- Lag compensator TF

$$G_C(s) = \frac{s+\frac{1}{T}}{s+\frac{1}{\alpha T}}$$

  - $\alpha > 1$

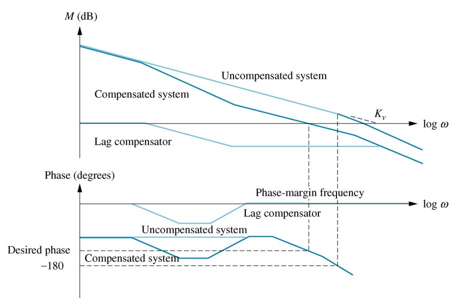

Figure: Visualizing lag compensation

Example, [1, p. 632]

Example (Lag compensation design)

- *Problem:* Use Bode diagrams to design a lag compensator to yield a tenfold improvement in steady-state error over the gain-compensated system while keeping $\%OS = 9.5\%$
- *Solution:* On the board

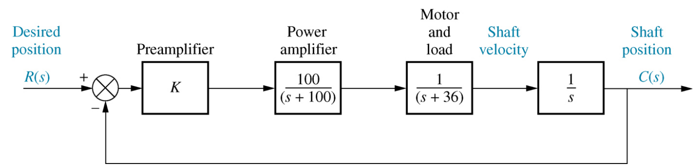

Figure: System

11 Design via frequency response
11.1 Introduction
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Visualizing lead compensation, [1, p. 635]

Concept

- Change the phase diagram
- $\uparrow$ gain crossover $\propto \uparrow$ bandwidth
- $\uparrow \Phi_M \propto \downarrow \%OS$
- $\uparrow \Phi_M \propto \downarrow T_p$
- Implement a steady-state error requirement $\rightarrow$ design a TR

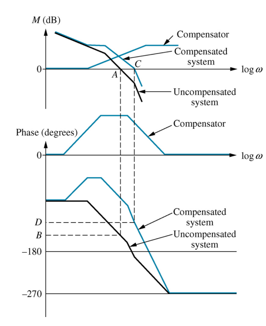


Figure: Visualizing lead compensation

Visualizing lead compensation, [1, p. 635]

Concept

- Lead compensator TF

$$G_C(s) = \frac{1}{\beta}\frac{s+\frac{1}{T}}{s+\frac{1}{\beta T}}$$

  - $\beta > 1$
- Frequency at maximum phase shift angle

$$\omega_{\text{max}} = \frac{1}{T\sqrt{\beta}}$$

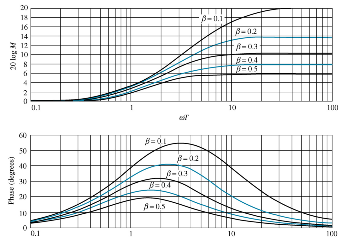


Figure: FR of a lead compensator for various values of β

Visualizing lead compensation, [1, p. 635]

Concept

- Maximum phase shift angle

$$\phi_{\text{max}} = \tan^{-1}\left(\frac{1-\beta}{2\sqrt{\beta}}\right) = \sin^{-1}\left(\frac{1-\beta}{1+\beta}\right)$$

- Magnitude at maximum phase shift angle

$$|G(j\omega_{\text{max}})| = \frac{1}{\sqrt{\beta}}$$

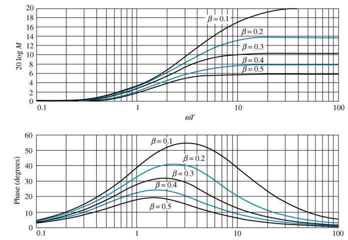


Figure: FR of a lead compensator for various values of β

Visualizing lead compensation, [1, p. 637]

Procedure

1. Find the CL bandwidth required to meet a T_s, T_p, or T_r requirement [1, p. 582]

$$\omega_{BW} = \omega_n\sqrt{(1-2\zeta^2)+\sqrt{4\zeta^4-4\zeta^2+2}}$$

$$\omega_n = \frac{4}{T_s\zeta} \quad \text{and} \quad \omega_n = \frac{\pi}{T_p\sqrt{1-\zeta^2}}$$

2. Since the lead compensator has negligible effect at low frequencies, set the gain, K, of the uncompensated system to the value that satisfies the steady-state error requirement
3. Plot the Bode diagrams for this value of gain and determine the uncompensated system's Φ_M

Visualizing lead compensation, [1, p. 637]

Procedure

4. Find the Φ_M to meet ζ or $\%OS$ requirement. Then evaluate the additional phase contribution required from the compensator.
5. Determine the value of β from the lead compensator's required phase contribution

$$G_C(s) = \frac{1}{\beta}\frac{s+\frac{1}{T}}{s+\frac{1}{\beta T}}$$

$$\phi_{\text{max}} = \tan^{-1}\left(\frac{1-\beta}{2\sqrt{\beta}}\right) = \sin^{-1}\left(\frac{1-\beta}{1+\beta}\right)$$

6. Determine the compensator's magnitude at the peak of the phase curve

$$|G(j\omega_{\text{max}})| = \frac{1}{\sqrt{\beta}}$$

Visualizing lead compensation, [1, p. 637]

Procedure

7. Determine the new ω_{Φ_M} by finding where the uncompensated system's magnitude curve is the negative of the lead compensator's magnitude at the peak of the compensator's phase curve
8. Design the lead compensator's break frequencies to find T and the break frequencies

$$G_C(s) = \frac{1}{\beta}\frac{s+\frac{1}{T}}{s+\frac{1}{\beta T}}$$

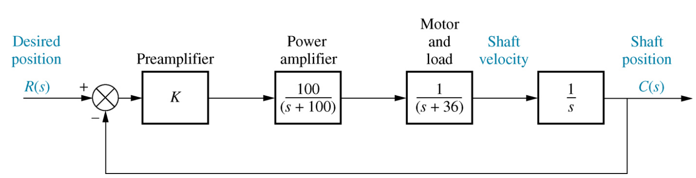
$$\omega_{\text{max}} = \frac{1}{T\sqrt{\beta}}$$

9. Reset the system gain to compensate for the lead compensator's gain
10. Check the bandwidth to be sure the speed requirement has been met
11. Simulate to be sure all requirements are met
12. Redesign if necessary to meet requirements

Example, [1, p. 638]

Example (Lead compensation design)

- *Problem:* Design a lead compensator to yield $\%OS = 20\%$, $K_V = 40$, & $T_p = 0.1$ second
- *Solution:* On the board



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- 11 Design via frequency response
  - 11.1 Introduction
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Intro, [1, p. 641]

Concept

- What we are not doing: separate lag & lead compensators
  1. Design a lag compensator to lower the high-frequency gain, stabilize the system, & improve the steady-state error
  2. Design a lead compensator to meet the phase-margin requirements
- What we are doing: passive lag-lead network
  - Eliminates the buffer amplifier that separates the lag network from the lead network

Visualizing lag-lead compensation, [1, p. 641]

Concept

- Lag-lead passive compensator TF

$$G_C(s) = G_{\text{Lead}}(s)G_{\text{Lag}}(s) = \left(\frac{s+\frac{1}{T_{\text{Lead}}}}{s+\frac{\gamma}{T_{\text{Lead}}}}\right)\left(\frac{s+\frac{1}{T_{\text{Lag}}}}{s+\frac{1}{\gamma T_{\text{Lag}}}}\right)$$

  - 1^{st} term in parentheses: lead compensator
  - 2^{nd} term in parentheses: lag compensator
  - γ replaces β & α of lead & lag networks, respectively

$$G_C(s) = \frac{1}{\beta}\frac{s+\frac{1}{T}}{s+\frac{1}{\beta T}} \quad G_C(s) = \frac{s+\frac{1}{T}}{s+\frac{1}{\alpha T}}$$

  - β & α must be reciprocals of each other
  - $\beta > 1$ & $\alpha > 1 \to \gamma > 1$

Visualizing lag-lead compensation, [1, p. 643]

Procedure

1. Using a 2^{nd}-order approximation, find the CL bandwidth required to meet T_s, T_p, or T_r [1, p. 582]
2. Set the gain, K, to the value required by the steady-state error specification
3. Plot the Bode diagrams for this value of gain
4. Using a 2^{nd}-order approximation, calculate the Φ_M to meet the ζ or $\%OS$ requirement [1, p. 590]
5. Select a new ω_{Φ_M} near ω_{BW}
6. At the new ω_{Φ_M}, determine the additional amount of phase lead required to meet the Φ_M requirement. Add a small contribution that will be required after the addition of the lag compensator.

Visualizing lag-lead compensation, [1, p. 643]

Procedure

7. Design the lag compensator by selecting the higher break frequency one decade below the new ω_{Φ_M}. The design of the lag compensator is not critical, and any design for the proper Φ_M will be relegated to the lead compensator. The lag compensator simply provides stabilization of the system with the gain required for the steady-state error specification. Find the value of γ from the lead compensator's requirements. Using the phase required from the lead compensator, the phase response curve can be used to find the value of $\gamma = \beta^{-1}$. This value, along with the previously found lag's upper break frequency, allows us to find the lag's lower break frequency.

Visualizing lag-lead compensation, [1, p. 643]

Procedure

8. Design the lead compensator. Using the value of γ from the lag compensator design and the value assumed for the new ω_{Φ_M}, find the lower- and upper-break frequency for the lead compensator.

$$\omega_{\text{max}} = \frac{1}{T\sqrt{\beta}}$$

9. Check the bandwidth to be sure the speed requirement has been met
10. Redesign if Φ_M or TR specifications are not met, as shown by analysis or simulation

Example, [1, p. 643]

Example (Lag-lead compensation design)

- *Problem:* Design a passive lag-lead compensator using Bode diagrams to yield $\%OS = 13.25\%$, $T_p = 2$ seconds, & $K_v = 12$

$$G(s) = \frac{K}{s(s+1)(s+4)}$$

- *Solution:* On the board

Bibliography

Norman S. Nise. *Control Systems Engineering*, 2011.